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The Extent of Use of Concrete-Representational-Abstract (CRA) Model in Mathematics

Nelsa Gallo-Toong, EdD

Abstract

This study aimed to determine the extent of use of CRA model in Mathematics. The schools covered by this study were the 5 secondary schools in the Division of Negros Occidental of School Year 2015 – 2016. There were 267 Grade 8 students who were taken as respondents of the study. The study was descriptive and correlational in nature. The research utilized mean, weighted mean and Pearson Product- Moment Coefficient of Correlation. The study revealed that the extent of teachers' use of CRA model in addition and subtraction of integers, multiplication and division of integers, polynomials, linear equations in one unknown, factoring, addition of rational numbers, algebraic word problem and angles as perceived by the students was "high". Likewise, the study further revealed that the academic performance of more or less one-fourth of the students in Mathematics was in the "developing" level.

Keywords: *Concrete-Representational-Abstract, academic performance*

Introduction

Every person who enters the teaching profession has dreams of changing the lives of thestudents for the better. Teachers have limitless possibilities when it comes to the methods ofinstruction that they can use. Teachers must acknowledge that every student is different. Whilesome students learn from visual clues as in pictures or videos, others learn from hands onexperience.

Mathematical ideas are abstract mental constructs. To help students grasp these ideas,they must be represented in a more concrete way using external representations. Theseexternal representations take the place of the abstract, mental concepts, and they embody thekey properties of the concepts.

The ideas of just verbalizing the rules and principles in Mathematics and presentingexamples and solutions on the chalk board have undergone improvisations. An over-dependence of the chalk-and-talk method has resulted in unsatisfactory performance and poorattitude towards school Mathematics for many students.

In addition, many students, despite a good understanding of Mathematical concepts,make errors because they misread signs or carry numbers incorrectly for one reason that theyare not adept to visualize what are really given and required in the problem (Lauren and Lee,2012).

To assist students in acquiring a conceptual understanding of Mathematics,severalstrategies have been developed. One such strategy is the Concrete-Representational-Abstractmethod of instruction. It is teaching students through theuse of concrete objects, pictorialrepresentations, then abstract numerals. In thisstrategy, each student learns differently andteachers will be better able to reach allthe students in their classroom. This method of instruction assists students in developing a conceptual understanding of Mathematical skills andrelationships (Flores et al., 2014; Mancl et al., 2012).

According to the Standards for Mathematical Practice, "Mathematically proficient students start by explaining to themselves the meaning of a problem" (McCallum, 2011). Without physically seeing something or applying it to real-life, students struggle to make sense of Mathematics problems. In addition, McGahan, ,(2014), stressed that one method to guide students in this sense-making process is Mathematical modeling. An example of Mathematical modeling is using concrete models, which are tangible objects that aid in the connection between Mathematics concepts and abstract symbols. With a hands-on approach in the classroom, students can grasp what the problems actually mean. They see why something is happening, which hopefully gives meaning to the problems and leads to a deeper understanding of the material. Using concrete models is interactive and collaborative, and brings a different, primarily student-based teaching method into the classroom.

The researcher believes that the teacher has an important role in the Mathematics achievement of the students. It was this reality that urged the researcher to investigate the use of Concrete-Representational-Abstract (CRA) model to students' performance in Mathematics. This information would also be very useful to aid Mathematics teachers in using CRA as an intervention in Mathematics instruction and could be a variable to improve students' academic performance.

Research Design

The researcher used the descriptive–correlational methodology of research because it aimed to describe the extent of teachers' use of Concrete- Representational-Abstract (CRA) model and determine if this had a relationship to students' academic performance.

Survey questionnaires for the extent of teachers' use of CRA model as viewed by the students were made through the different chosen topics in the Grade 7 lessons. These were rated by the students to measure their teachers' extent of the use of CRA approach in their Mathematics subject. The class academic performance in Mathematics in school year 2014-2015 was also gathered. The extent of teachers' use of CRA model as perceived by the students was used to correlate with students' academic performance in Mathematics.

Research Environment

The place of the study is in the Municipality of Hinoba-an, the last municipality in Negros Occidental which has 13 barangays. Some have no access of electricity especially those part of barangays located in the highland. Although other barangays have electricity, yet there are no internet connections. Water supply among other barangays is not sufficient to cater to the everyday needs of the people.

The study focused specifically on the secondary schools in the District of Hinoba-an, Division of Negros Occidental. It has 3 public schools, with 1 extension school and 2 annexed schools.

Research Respondents

The respondents of the study were the 267 Grade 8 students in the District of Hinoba-an, Division of Negros Occidental. The respondents of the study were chosen using the systematic sampling method in which every 3rd student in the list was selected.

Research Instruments

The study made use of self-made Mathematics problems and some problems related to CRA presentations which were found in the internet. The researcher also considered the study of Shaunita D. Strozier (2012) on “Math Gains: Algebra Tiles and The Effects of Concrete-Representational- Abstract Sequence and a Mnemonic Strategy on Algebra Skills of Students Who Struggle in Math.”

To ensure the validity of the instrument and to avoid culture bias, the problems were evaluated by the three experts. These experts are all master’s degree holders with specialization in Mathematics. Two of them were teaching in the tertiary level and the other one is a high school district Mathematics coordinator. Some of the items, instructions, and illustrations were revised after the comments and suggestions by the experts. All items were considered by them but some terms were edited.

To ensure item reliability, a dry-run was made. There were 30 selected students who served as the respondents. These students were not the actual respondents. The items were tested for its reliability using the Cronbach’s alpha test. This was calculated to verify the internal consistency reliability of the items. It is a measure of the extent to which all the variables in the scale are positively related to each other and its theoretical value varies from 0 to 1. Higher values of alpha are more desirable and a value of 0.70 is considered acceptable. According to McMillan and Schumacher, this test is regarded as the most suitable type of survey research where items are not scored right or wrong and where each item could have different answers (cited in Feril, 2014). The reliability coefficient yielded a value of 0.846. This means that the items were reliable. Then the final revision of the questionnaires was done.

Research Procedure

After the design hearing, the researcher wrote a letter of request for the distribution of the final questionnaires noted by the dean of the graduate school and signed by the Schools Division Superintendent in the Division of Negros Occidental. The signed and approved request was presented to the secondary school teachers in the District of Hinoba-an, Division of Negros Occidental. During the distribution of the questionnaires, the researcher explained to the respondents that the test that they would be answering would not influence their performance as students and that data collected would remain confidential and would only be used for the purposes of the study. The retrieval of the questionnaires was done right after they had answered the questions.

The grades of the students were gathered through the permission of the principal.

Findings

Table 1. Extent of Teachers' Use of CRA Model in Teaching Addition and Subtraction of Integers as Perceived by the Students

Topic	$w\bar{x}$	Verbal Description	Extent of CRA Use
Addition of Integers			
1. Addition of positive and negative integers. {Ex: Find the sum of (+5) and (-6). }	4.07	Frequently	High
2. Addition of positive integers. {Ex: Find the sum of (+2) and (+3).}	4.01	Frequently	High
Subtraction of Integers			
3. Subtraction of integers with lesser subtrahend. {Ex: What is the difference when (+3) is subtracted from (+5)? }	3.80	Frequently	High
4. Subtraction of integers with greater subtrahend. {Ex: What is (+2) – (+4)? }	3.87	Frequently	High

Table 1 reveals that the extent of teachers' use of Concrete-Representational-Abstract (CRA) Model is "high" in teaching addition and subtraction of integers. This means that they use the sequence 61%-80% of the time in presenting these areas. As reflected in the study of Estoconing (2015), these two topics generally were revealed to be"moderately difficult"and "difficult", respectively. Hence, strategic presentation of the concepts must be observed. Badarudin (2008) who found these topics "difficult" stressed that understanding the concepts well requires the students to build mental images and models that allow them to visualize these new comparisons and relationships.Bolyard and Moyer-Packenham (2006) likewise pointed out in their study that the use of virtual manipulatives in integer instruction indicates a positive gain in achievement, however, they do not develop a complete understanding of integer addition and subtraction concepts. Hill (2008) added that using integer tiles gave students opportunities to create concrete, visual, and symbolic models of integer addition. Manipulating the tiles visually showed what happened when two same-sign or different-sign integers were added. By working together, students were able to verbalize and discuss the process of adding same-sign and different-sign integers, which helped students understand the process of integer addition. This method proved to be more successful than the use of drill-and-practice exercises without manipulatives, which had been the method used in previous years with this unit.

Table 2. Extent of Teachers' Use of CRA Model in Teaching Multiplication and Division of Integers as Perceived by the Students

Topic	$w\bar{x}$	Verbal Description	Extent of CRA Use
Multiplication of Integers			
5. Multiplication of positive and negative integers. {Ex: Give the product of (-2) and (+4). }	3.79	Frequently	High
6. Multiplication of negative integers or integers with like sign. {Ex: What is (-2)(-3)? }	3.78	Frequently	High
Division of Integers			
7. Division of positive integers. {Ex: Divide (+12) ÷ (+3). }	3.60	Frequently	High

The data in Table 2 reflect that the extent of teachers' use of CRA model in teaching multiplication and division of integers is "high". This means that the utilization of the sequence is between 61%-80%. This extent should be increased as Akyuz(2012) revealed that negative integers create difficulties to students. They try to make sense of them based on what they learn about natural numbers and assume that what they know about natural numbers also holds true for integers.

Table 3. Extent of Teachers' Use of CRA Model in Teaching Polynomials as Perceived by the Students

Topic	$w\bar{x}$	Verbal Description	Extent of CRA Use
Translation of Polynomials			
8. Translation of verbal phrase into mathematical phrase. (Ex: Translate the square of a number decreased by four of a number.)	3.70	Frequently	High
Addition of Polynomials			
9. Addition of polynomials with three terms. (Ex: Add $x^2 + 3x – 1$ and $x^2 – 2x – 3$.)	3.51	Frequently	High
Multiplication of a Polynomial by a Monomial			
10. Multiplication of a monomial to a binomial. {Ex: Multiply $2(x^2 + 2)$.}	3.73	Frequently	High
Multiplication of Binomial			
11. Multiplication of two binomials. {Ex: Find the value of (2x + 3)(x + 4).}	3.73	Frequently	High
Division of a Polynomial by a Monomial			
12. Division of a binomial to a monomial (Ex: Find the value of $\frac{2x^2+4}{2}$).	3.88	Frequently	High

Table 3 presents that theextent of teachers' use of CRA model in teaching polynomials in the following areas is "high": translation and addition of polynomials; multiplication of a polynomial by a polynomial; multiplication of binomial; and division of a polynomial by a monomial. Results in a study indicated that the integration of the concrete manipulatives, sketches of manipulatives, and abstract notation with the support of a graphic organizer was an effective strategy to improve students' conceptual understanding and procedural fluency of multiplying two linear expressions (Strickland, T. K., & Maccini, P. ,2012).

Table 4. Extent of Teachers' Use of CRA Model in Teaching Linear Equations in One Unknown, Factoring and Addition of Rational Numbers as Perceived by the Students

Topic	$w\bar{x}$	Verbal Description	Extent of CRA Use
Linear Equations in One Unknown			
13. Linear equations in one unknown with two terms in both sides of the equation. (Ex: Solve the equation 2x + 1 = x + 6).	3.65	Frequently	High
Factoring			
14. Factor trinomial with *a* = 1. {Ex: Factor ($x^2 + 5x + 6$).}	3.68	Frequently	High
Addition of Rational Numbers in Fraction Form			
15. Addition of similar fractions. (Ex: Find the sum of 2/4 + 1/4 .)	3.91	Frequently	High

Table 4 depicts that the extent of teachers' use of CRA model in teaching linear equations in one unknown, factoring and addition of rational number is "high" which means the utilization is between 60%-81%. The study of Suyat-Ablanrevealed that grade six pupils found rational numbers as one of the most difficult topics (quoted by Bohol, 2013). According to Bohol (2013), if a student understands fractions, then he/she can understand any Mathematics concept. It is then very important for every Mathematics teacher to know how to teach fractions in the most approachable way possible. Rivera (2010) used Algeblocks to teach students about factoring polynomials. Using the Algeblocks, the students understood the process of factoring better. The visual process helped them appreciate the algebraic process and understand the symbols they were manipulating.

Table 5. Extent of Teachers' Use of CRA Model in Teaching Algebraic Word Problem and Angles as Perceived by the Students

Topic	$w\bar{x}$	Verbal Description	Extent of CRA Use
Algebraic Word Problem			
16. Solve word problem related to number relations. (Ex: Martin picked 24 apples. He picked four times as many apples as Julia. How many apples did Julia pick?)	3.61	Frequently	High
Angles			
17. Find the measure of an angle using the concept of vertical angles. {Ex: Angles A and D are vertical angles. If <A = (20 – 2m)º and <D = (m – 130)º, what is the value of m?}	3.77	Frequently	High

Table 5 indicates that the extent of teachers' use of CRA model in teaching algebraic word problem and angles as perceived by the students is"high". Mancl(2011) identified some factors that contribute to word problem challenges and one of these is the inability to comprehend the sentence especially if extraneous information in the problem is given. He also mentioned that a combined word problem strategy that incorporates a Concrete-Representational-Abstract instructional sequence, schema-based diagrams, and a cognitive strategy may be used to improve students' ability to solve word problems (cited in Pagbonocan, 2015).

Table 6. Academic Performance of the Students in Mathematics

Grade	Verbal Equivalent	Frequency	Percent
90% and above	Advanced	33	12.36
85% - 89%	Proficient	73	27.34
80% - 84%	Approaching Proficiency	91	34.08
75% - 79%	Developing	70	26.22
Total		267	100.00

Note: Average Performance = 83.27 (Approaching Proficiency)
Standard Deviation = 4.98

Table 6 reveals that the average performance of the students is 83.27% and is described to be in the "approaching proficiency" level.The student at this level has developed the fundamental knowledge and skills and core understandings, and with little guidance from the teacher and/or with some assistance from peers, and can transfer these understandings through authentic performance tasks.

Table 7. Relationship between the Extent of Teachers' Use of CRA Model and the Academic Performance of the Students

Variables	Computed r	Decision	Remark
Teacher's Extent of Use of CRA Model and the Academic Performance of the Students	0.033	Do not Reject H_o	Not Significant

Tabular r = 0.120; df = 265; α = 0.05

The data in Table 7 reveal that the computed "r" value (0.033) is less than the tabular "r" value (0.120). This finding will not allow rejection of the null hypothesis. This means that the data gathered are not sufficient to conclude that relationship exists between the extent of teachers' use of CRA Model and the academic performance of the students.This result do not conform with the study of Hill (2008) indicates that students who learn operations using manipulatives outperform students who do not, as long as the teacher is knowledgeable about the manipulatives and their connection with the symbolic Mathematical representation (cited in Estoconing, 2015).

Conclusions

Based on the findings cited above, the following conclusions are hereby drawn:

1. The extent of teachers' use of CRA model isgenerally "high" in all areas in Mathematics considered in this study.

2. The study indicated that the mean academic performance of the students is in the "approaching proficiency" level. However, more or less one-fourth of them are in the "developing" level.

3. The data are not sufficient to indicate significant relationship between the extent of teachers' use of CRA model and the academic performance of the students.

In general, the extent of teachers' use of CRA in teaching Mathematics is"high".

Recommendations

In the light of the findings of this study and the conclusions drawn, the following recommendations are suggested:

1. Teachers should properly follow the sequence in usingthe CRA in teaching Mathematics to increase the result from "high" to "very high" utilization.

2. Teachers should devise a strategy appropriate to reinforce students classified in the "developing" level. They should also come up with an approach to solidify students categorized in the "approaching proficiency" level and better.

3. An experimental study on the use of CRA be conducted to identify its effect.

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Appendix

Survey Instrument

The Extent of Use of Concrete-Representational-Abstract (CRA) Model in Mathematics

This questionnaire aims to identify relationship of the extent of use of CRA in teaching mathematics in relation to students' academic performance. Kindly and honestly answer the following questions. It is assured that the information you share is confidential. Thank you very much for your time and cooperation.

Name: ____________________________ **School:**___________________

Direction: 1. Read each statement. Think carefully about each statement and respond as truthfully as you can.
2. Place a check mark (√) in the blank that best describes your teachers' use of CRA. The following will be your guide.

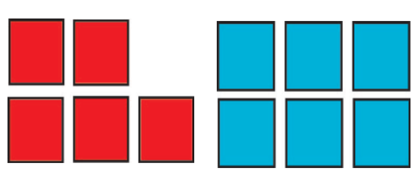
Verbal Description	Explanation
Almost Always	The CRA sequence is applied by the teacher 81% - 100% of the time.
Frequently	The CRA sequence is applied by the teacher 61% - 80% of the time.
Sometimes	The CRA sequence is applied by the teacher 41% - 60% of the time.
Rarely	The CRA sequence is applied by the teacher 21% - 40% of the time.
Almost Never	The CRA sequence is applied by the teacher 1% - 20% of the time.

A. Topic: Addition of Integers

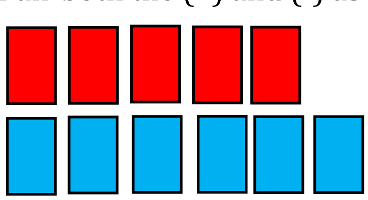
A.1. Addition of positive and negative integers.

Example 1: Find the sum of (+5) and (-6).

- **Concrete:** Use two sets of algebra tiles. Represent positive integers with red tiles and negative integers with blue tiles. One red tile and one blue tile is equivalent to 0.Pair the red and blue tile. What is left?



- **Representational:** Draw the tiles on the board. Pair both the (+) and (-) as 0. What is left?



A.1. Practiced / used by my teacher

_____ Almost Always
_____ Frequently
_____ Sometimes
_____Rarely
_____ Almost Never

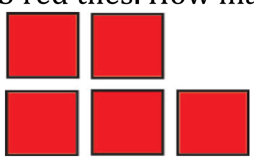
- **Abstract:** Uses the concept of drawings into symbols. In adding numbers having unlike sign, subtract the numbers and copy the sign with greater absolute value.

  (+5) + (-6) = -1

A.2. Addition of positive integers.

Example 2: Find the sum of (+2) and (+3).

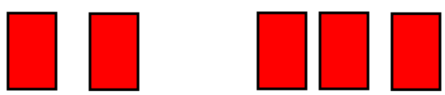
- **Concrete:** Use algebra tiles. Represent positive integers with red tiles. Two red tiles and another 3 red tiles. How many red tiles are there?



A.2. Practiced / used by my teacher

____ Almost Always
____ Frequently
____ Sometimes
____Rarely
____ Almost Never

- **Representational:** Draw the tiles on the board. How many positive tiles are there?



- **Abstract:** Uses the concept of drawings into symbols. In adding numbers having like sign, add the numbers and copy the common sign.

  (+2) + (+3) = (+5)

B. Topic: Subtraction of Integers

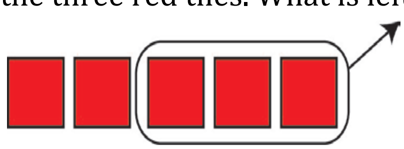
B.1. Subtraction of integers with lesser subtrahend

Example 1: What is the difference when (+3) is subtracted from (+5)?

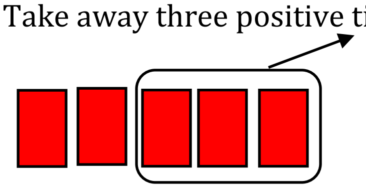
B.1. Practiced / used by my teacher

____ Almost Always
____ Frequently
____ Sometimes
____Rarely
____ Almost Never

- **Concrete:** Use algebra tiles or colored chips. Represent positive integers with red tiles. Take away the three red tiles. What is left?



- **Representational:** Draw the tiles on the board. Take away three positive tiles. What is left?



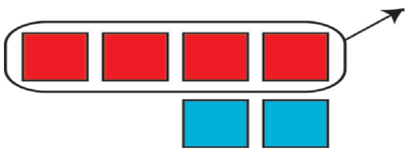
Abstract: Use the concept of drawings into symbols. Get the additive inverse of the subtrahend and proceed in adding integers.

(+5) – (+3) = (+5) + (-3) = (+2)

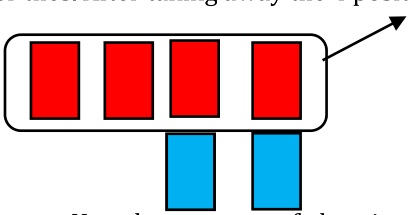
B.2. Subtraction of integers with greater subtrahend.

Example 2: What is (+2) – (+4)?

- **Concrete:** Use algebra tiles or colored chips. Represent positive integers with red tiles. Use two red tiles and since u need to take away 4 red tiles, you need to add another 2 red tiles. Remember that whatever you added, you do the same using the other tiles. After taking away the 4 red tiles, what is left?



- **Representational:** Draw the tiles on the board. Make two positive tiles with a positive sign. Since u need to take away 4 positive tiles, you need to add another 2 positive tiles. Remember that whatever you added, you do the same using the other tiles. After taking away the 4 positive tiles, what is left?



B.2.Practiced / used by my teacher

____ Almost Always
____ Frequently
____ Sometimes
____Rarely
____ Almost Never

- **Abstract:** Use the concept of drawings into symbols. Get the additive inverse of the subtrahend and proceed in adding integers.

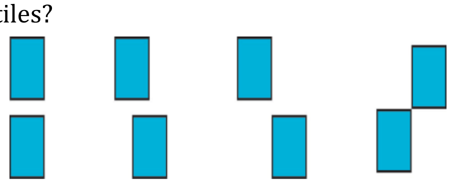
(+2) – (+4) = (+2) + (-4) = (-2)

C.Topic: Multiplication of Integers

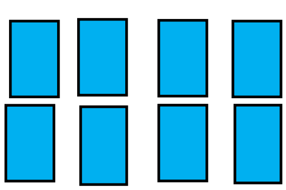
C.1. Multiplication of positive and negative integers.

Example: Give the product of (-2) and (+4)

- **Concrete:** Use algebra tiles or colored chips. Represent negative integers with blue tiles. It means adding four groups of two. How many tiles are there? What kind of tiles?



- **Representational:** Draw tiles on the board. Pair it in order to have four pairs. How many tiles are there? What kind of tiles?



C.1. Practiced / used by my teacher

____ Almost Always
____ Frequently
____ Sometimes
____Rarely
____ Almost Never

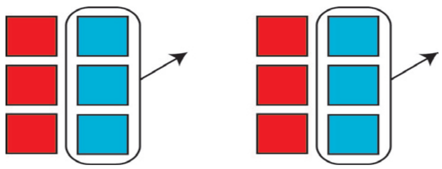
- **Abstract:** Use the concept of drawings into symbols. Multiply the numbers and count the number of a negative sign. If it is odd, the product is positive. If it is even, the product is positive.

  (-2) (4) = (-8)

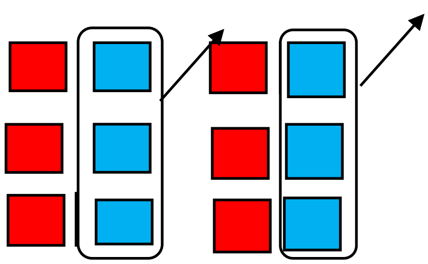
C.2. Multiplication of negative integers or integers with like sign.

Example 2: What is (-2)(-3)?

- **Concrete:** Use algebra tiles or colored chips. Represent negative integers with blue tiles. Since we have the same sign,we would start with six red tiles and six blue tiles (which together have a value of zero), then we would take away two groups of three blue tiles. What is left?



- **Representational:** Draw tiles on the board. Since we have the same sign,we would start with six positive tiles and six negative tiles (which together have a value of zero), then we would take away two groups of three negative tiles. What is left?



- **Abstract:** Use the concept of drawings into symbols. Multiply the numbers and count the number of a negative sign. If it is odd, the product is positive. If it is even, the product is positive.

  (-2) (-3) = (+6)

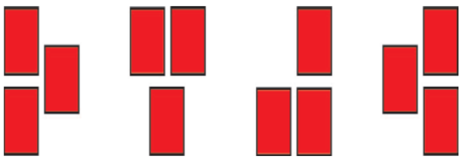
C.2.Practiced / used by my teacher

____ Almost Always
____ Frequently
____ Sometimes
____Rarely
____ Almost Never

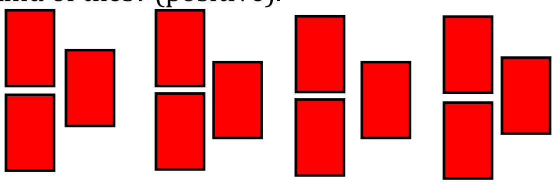
D. Topic: Division of Integers

Division of positive integers.
Example: Divide (+12) ÷ (+3)

- **Concrete:** Use algebra tiles or colored chips. Represent negative integers with red tiles. Group the tiles by three. How many groups are there? (4).What is the color of the tiles? (red which represent positive).



- **Representational:** Draw tiles on the board. Group it by three. How many groups were formed? (4). What kind of tiles? (positive).



- **Abstract:** Translate the concept of drawings into symbols. Dividing 12 by 3 will give 4. Since they are like signs, the quotient is positive.

(+12) ÷ (+3) = (+4)

D.Practiced / used by my teacher

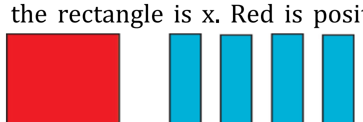
_____ Almost Always
_____ Frequently
_____ Sometimes
_____Rarely
_____ Almost Never

E. Topic: Polynomials

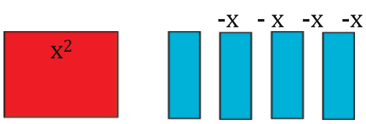
Translation of verbal phrase into mathematical phrase.

Example: Translate the square of a number decreased by four of a number.

- **Concrete**: Use colored chips or algebra tiles. The square represent x^2 and the rectangle is x. Red is positive while blue is negative.



- **Representational:** Draw on the board a square to represent the square of x and a rectangle to represent –x.



- **Abstract:** Translate the concept of drawings intosymbols. The square of a number decreased by the product of four and a number = $x^2 - 4x$.

E.Practiced / used by my teacher

_____ Almost Always
_____ Frequently
_____ Sometimes
_____Rarely
_____ Almost Never

F. Topic: Addition of Polynomials

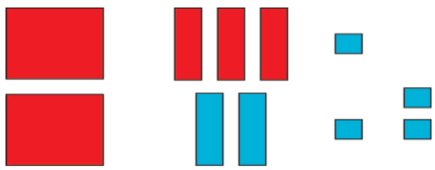
Addition of polynomials with three terms.

Example: Add $x^2 + 3x - 1$ and $x^2 - 2x - 3$.

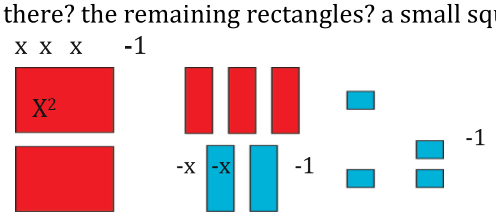
F.Practiced / used by my teacher

_____ Almost Always
_____ Frequently
_____ Sometimes
_____Rarely
_____ Almost Never

- **Concrete:** Use colored chips or algebra tiles. The big square tile represents x^2, a rectangle tile is x and the small square tile as unit (1). Red tile is positive while the blue one is negative. A pair of the same shapes but of different colors is equivalent to zero. Find out how many big red squares are there? (2 or $2x^2$). How many remaining rectangles are there after pairing? (1 red which means positive x). How many small squares? (4 and since blue tile represents negative, so it is -4).



- **Representational:** Draw big and small squares and a rectangle. Represent big squares as x^2, small squares as unit (1) and a rectangle as x. A pair of a positive and a negative sign is equal to zero. How many big squares are there? the remaining rectangles? a small square?



*

Abstract: Use symbols. Combine terms with the same literal coefficients or similar terms. Observe the rule of integers.

$(x^2 + 3x - 1) + (x^2 - 2x - 3) = (x^2 + x^2) + (3x - 2x) + (-1 - 3) = 2x^2 + x - 4$

G. Topic: Multiplying a Polynomial by a Monomial

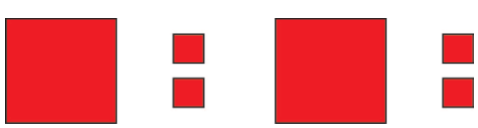
Multiplication of a monomial to a binomial.

Example: Multiply $2(x^2 + 2)$.

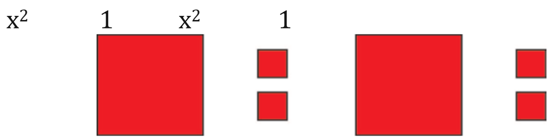
G.Practiced / used by my teacher

_____ Almost Always
_____ Frequently
_____ Sometimes
_____Rarely
_____ Almost Never

- **Concrete:** Use colored chips or algebra tiles in presenting the concept. Represent the big square as x^2 and the small square as 1. The red color means positive. $2(x^2 + 2)$ means adding two groups of $x^2 + 2$. Find out how many are big red squares? Red small squares?



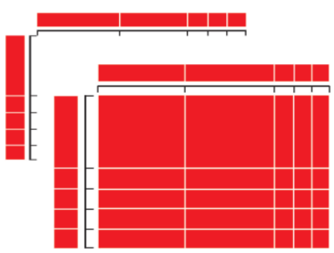
- **Representational:** Draw big square and a small square on the board. Represent the big square as x^2 and a small square as 1. $2(x^2 + 2)$ means adding two groups of $x^2 + 2$. Find out how many big squares as x^2 are there? Small squares as 1 are there?
  There are 2 big squares as x^2 and 4 small squares as 1.



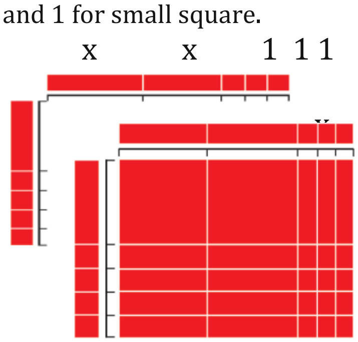
- **Abstract:** Use symbols. Multiply the monomial times each of the terms of the polynomial.
  $2(x^2 + 2) = 2(x^2) + 2(2) = 2x^2 + 4.$

H. Topic: Multiplication of Binomial
Multiplication of two binomials.
Example: Find the value of $(2x + 3)(x + 4)$

- **Concrete:** Use the colored chips or algebra tiles in presenting the concept. Represent the rectangle as x and the square as unit (1). The red color means positive. Make a rectangle with dimension would be $(2x + 3)$ and the other would be $(x + 4)$. Complete the rectangle. Find out what happened? What completed the rectangle? How many are big squares? Small squares? Rectangles? A big square represents x^2, x for rectangle and unit (1) for small square.



- **Representational:** Draw a rectangle with dimension would be $(2x + 3)$ and the other would be $(x + 4)$. Complete the rectangle. Find out what happened. What completed the rectangle? How many are big squares? Small squares? Rectangles? A big square represents x^2, x for rectangle and 1 for small square.



H.Practiced / used by my teacher

____ Almost Always
____ Frequently
____ Sometimes
____Rarely
____ Almost Never

- **Abstract:** Use symbols. Multiply every term in one binomial by every term in the other. Multiply the first terms, outside terms, inside terms and the last terms.

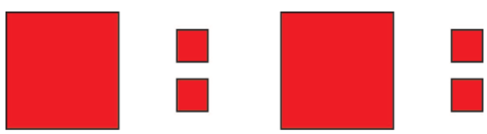
$$(2x + 3)\ (x + 4) = 2x^2 + 8x + 3x + 12$$
$$= 2x^2 + 11x + 12$$

I. Topic: Division of a Polynomial by a Monomial

Division of a binomial to a monomial

Example: Find the value of $\frac{2x^2+4}{2}$.

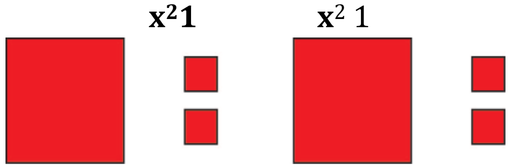
- **Concrete:** Use the colored chips or algebra tiles to represent the concept. Represent the big square as x^2 and the small square as unit (1). Red color means positive. Split the tiles into two equal groups. Find out how many x^2 and unit (1) are in each group? There is 1 big square and there are 2 small squares.



- **Representational:** Draw big square and a small square on the board. Represent the big square as x^2 and a small square as unit (1). $\frac{2x^2+4}{2}$ means dividing $2x^2 + 4$ into two equal groups. Find out how many big squares as x^2 are there? Small squares as unit (1) are there?

  There is 1 big square as x^2 and 2 small squares as unit (1).

$x^2$1 x^2 1



- **Abstract:** Use the symbols. Each term in the numerator is divided by the monomial in the denominator.

$$\frac{2x^2+4}{2} = \frac{2x^2}{2} + \frac{4}{2} = x^2 + 2$$

I. Practiced / used by my teacher

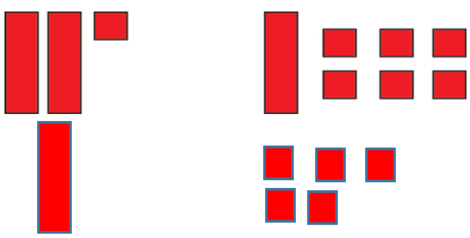
_____ Almost Always
_____ Frequently
_____ Sometimes
_____Rarely
_____ Almost Never

J. Topic: Solving Linear Equations in One Unknown

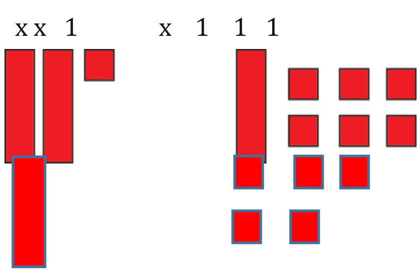
Linear equations in one unknown with two terms in both sides of the equation.

Example: Solve the equation 2x + 1 = x + 6

- **Concrete:** Use the colored chips or algebra tiles to represent the concept. Represent the rectangle tile as x and the small square tile as unit (1). Red color means positive. Divide the equation into two parts. The left side is the 2x + 1 and on the right side is x + 6. Assume that we wish the variable to be on the left side. What term is also on that side and how can it be removed? Get one rectangle tile for each side to isolate the variable on the right side. Get one small square tile on both sides to isolate 1 on the left side. Find out how many rectangle tiles are on the left and small square tiles are on the right side? There is only one rectangle tile on the left side and there are five small square tiles on the right side.



- **Representational:** Draw rectangles and a small square on the board. Represent the rectangle as x and a small square as 1. Arrange the drawing into two parts. The left side represents 2x + 1 and the right side is x + 6. Assume that we wish the variable to be on the left side. What term is also on that side and how can it be removed? Omit one rectangle for each side to isolate the variable on the right side. Omit one small square on both sides to isolate 1 on the left side. Find out how many rectangles are on the left and small squares on the right side? There is only one rectangle on the left side and there are five small squares on the right side.



- **Abstract:** Use symbols. Assume the variable to be on the left side. To remove the variable on the right side, subtract it to both sides of the equation. Assume the constant to be

J.Practiced / used by my teacher

_____ Almost Always
_____ Frequently
_____ Sometimes
_____Rarely
_____ Almost Never

on the right side. To remove the constant on the left side, subtract it to both sides of the equation.

$2x + 1 = x + 6$
$2x - x + 1 = x - x + 6$
$X + 1 = 6$
$X + 1 - 1 = 6 - 1$
$X = 5$

K. Topic: Factoring

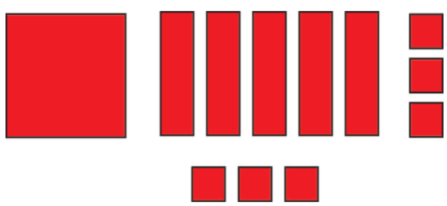
Factor trinomial with $a = 1$.

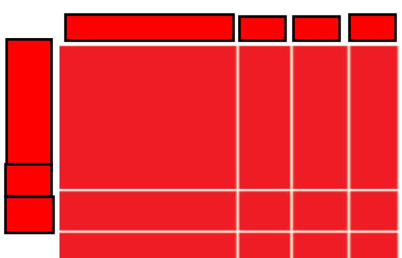
Example:Factor ($x^2 + 5x + 6$).

K.Practiced / used by my teacher

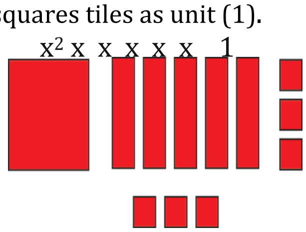
_____ Almost Always
_____ Frequently
_____ Sometimes
_____Rarely
_____ Almost Never

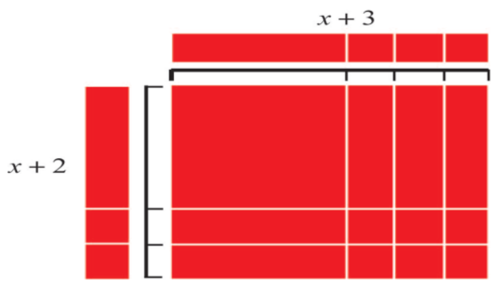
- **Concrete:** Use the colored chips or algebra tiles to represent the concept. Represent the big square tile as x^2, rectangle tile as x and the small square tile as unit (1). Red color means positive. Make a rectangular array of the tiles by placing big square tiles in the upper left and the unit (1) tiles in the lower right corner. Read the dimension of the completed rectangle. What is the dimension of a rectangle? The length has one rectangle tile and 2 small square tiles as unit (1). While the width has one rectangle tile and 3 small squares tile as unit (1).





- **Representational:** Draw tiles on the board. Represent the big square tile as x^2, rectangle tile as x and the small square tile as unit (1). Make a rectangular array of the tiles by placing big square tiles in the upper left and the unit (1) tiles in the lower right corner. Read the dimension of the completed rectangle. What is the dimension of a rectangle? The length has one rectangle tile and 2 small squares tiles as unit (1). While the width has one rectangle tile and 3 small squares tiles as unit (1).





- **Abstract:** Use symbols. Factor the first term and the last tem. Make sure to have the correct factors in the last term. To check the factors, the sum of the inner and outer product of the factors is the middle term of the given trinomial.

  ($x^2 + 5x + 6$)* pairs of factors of the last term
  6 and 1; 2 and 3*
  $(x + 3)(x + 2)$

L. Topic: Addition of Rational Numbers in Fraction Form

Addition of similar fractions.

Example: Find the sum of $\frac{2}{4} + \frac{1}{4}$.

- **Concrete:** Use plastic pies or fraction bars. Introduce how each part relates to the whole. Color the two parts in the first circle and one part of the second circle. How many colored region?

- **Representational:** Draw two circles. Divide each circle into four. Shade the two parts of the first circle and one part of the second circle. Count the number of shaded regions of both circles.

- **Abstract:** Use numbers and symbols. Explain what the numerator and denominator are. Present the rule in adding similar fractions. Add

L.Practiced / used by my teacher
____ Almost Always ____ Frequently ____ Sometimes ____Rarely ____ Almost Never

the numerator and copy the denominator. Reduce into simplest form if possible.

$$\frac{2}{4}+\frac{1}{4}=\frac{2+1}{4}=\frac{3}{4}$$

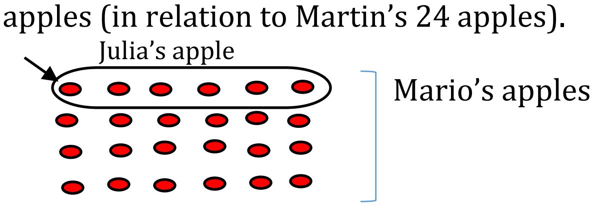
M. Topic: Algebraic Word Problem

Solve word problem related to number relations.

Example: Martin picked 24 apples. He picked four times as many apples as Julia. How many apples did Julia pick?

- **Concrete:** Use real apples, double-sided foam counters or other counters to represent the apples. Since the problem states that Martin picked four times as many apples as Julia, students may take Martin's 24 apples (the whole) and split them up into four equal groups. One group of 6 apples would then represent how many apples Julia picked.

- **Representational:** Represent the problem using red circles as apples to portray that Julia has 6 apples (in relation to Martin's 24 apples).



- **Abstract:** Explain how it will be solved using mathematical symbols and representations. Students may approach the problem as 4 x ? = 24. Realizing that this is really a division problem, they could then write 24 ÷ 4 = 6. Therefore, Julia picked 6 apples.

M.Practiced / used by my teacher

_____ Almost Always
_____ Frequently
_____ Sometimes
_____Rarely
_____ Almost Never

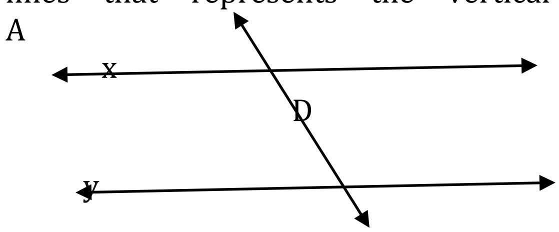
N. Topic: Angles

Find the measure of an angle using the concept of vertical angles.

Example: Angles A and D are vertical angles. If <A = (20 – 2m)º and <D = (m – 130)º, what is the value of ***m***?

- **Concrete:** Teach the concept using computers manipulative or concrete materials that will represent vertical angles.

- **Representational**: Draw on the board using lines that represents the vertical angles.



- **Abstract:** Use symbols and numbers. The idea that vertical angles are congruent, will result to angle A = angle B. Thus, 20 – 2m = m – 130. Combining similar terms, -2m – m = -130 -20.

-3m = -150

m = 50

N.Practiced / used by my teacher

_____ Almost Always
_____ Frequently
_____ Sometimes
_____Rarely
_____ Almost Never