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The Expected Value of Sampling Information for Linear Decision Problem on Two Actions under the Ga–E Model for Random Censoring Test

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Abstract: Firstly, this paper obtains the likelihood function based on the observed data under the random censoring test, then studies the posterior distribution of parameter of exponential distribution life for the Ga–E model under random censoring test, and focuses on the EVSI calculation formula of linear decision problem on two actions under the Ga–E model for random censoring test, however, most of the calculations are done using the Monte Carlo method.

Keywords: likelihood function; posterior distribution; revenue function; optimal action; Monte Carlo method

Mathematics Subject Classification: 62N01; 62F15; 62C10

1. Introduction

Decision-making problems are often encountered in people's daily life. For enterprises, the key to improve the management level of enterprises is to make correct decisions. There are some research results on linear decision problem, which can be found in references [1–10]. Two-action linear decision problem is a kind of simple and commonly used decision problem, which is often encountered in practice. In two-action decision problem, if the decision maker has the complete information, he can choose the best action and get the maximum benefit. But in the practical application, complete information is not known in advance, we can get more information through trial production, trial marketing and other means for the problem that needs to be decided. However, generally the obtained information is incomplete. In order to obtain more information and

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make the decision more reliable, sampling tests and other methods are often needed to add information to adjust the prior probability of the occurrence of the state, so that the adjusted prior probability is closer to the current objective reality. And there is a cost to these jobs. To this end, we need to study how much the benefit (or loss) of decision makers changes before and after adjusting the state probability with sampling information, that is, how much the decision makers can gain from sampling. This involves the expected value of sampling information (denoted as EVSI), which lays a foundation for further research on whether sampling is worth conducting. The EVSI calculation formula has been concluded for the normal distribution conjugate to the normal distribution decision model under the two-linear action. Exponential distribution is a very important life distribution. It is assumed that the life of a product follows exponential distribution in many occasions, such as the life of an electronic component, the talk time of a telephone, the service time in a random service system, etc. Exponential distribution is widely used in reliability. Therefore, in many cases, population X subject to exponential distribution should be considered, and the sampling distribution is $E(\lambda)$. It is generally assumed that the prior distribution of λ is a gamma distribution, and the posterior distribution of λ can be proved to be still a gamma distribution. In this paper, we discuss the decision model (namely Ga-E model) in which gamma distribution is conjugate to exponential distribution under two linear actions, and the calculation of EVSI under two linear actions (Ga-E) model.

The rest of this paper is organized as follows. In Section 2, we describe the random censoring test, and obtain the likelihood function based on the observed data. Section 3 discusses the Ga–E model under random censoring test, and obtains the posterior distribution of parameter of exponential distribution life. Posterior mathematical expectation is obtain by the Monte Carlo method. Section 4 describes the general form of linear decision model on two actions, and focuses on the optimal action. Section 5 obtains the EVSI calculation formula of linear decision problem on two actions under the Ga–E model for random censoring test, however, most of the calculations are done using the Monte

Carlo method. Finally, we summarize and conclude the paper in Section 6.

2. Random censoring test

Suppose the product life of the tested products $X_1, X_2, \cdots$ are the sequence of independent identically distributed non-negative random variables, the distribution function of X_i is $F(x\,|\lambda) = P(X_i \leq x)$, the density function is $f(x\,|\lambda)$, and λ is the parameter. The censored Life time $Y_1, Y_2, \cdots$ are a sequence of independent non-negative random variables, the distribution function of Y_i is $G_i(y)$, the density function is $g_i(y)$. Assume that $\{X_i\}$ is independent of $\{Y_i\}$. There are now n products for life test. Let the observed data be $\{Z_i\}, i = 1, 2, \cdots, n$. Each Z_i has the following values.

(1) When $X_i \leq Y_i$, the product fails before being censored. At this time, the exact value of product life is known, so $Z_i = X_i$;

(2) When $X_i > Y_i$, the product life is greater than the censored time. At this time, only the censored time is known, but the product life is not known, so take $Z_i = Y_i$.

To sum up, we know $Z_i = X_i \wedge Y_i = \min(X_i, Y_i)$.

Set

$$\alpha_i = \begin{cases} 1, X_i \leq Y_i \\ 0, X_i > Y_i \end{cases} \quad i = 1, 2, \cdots, n.$$

Let $\boldsymbol{z}, \boldsymbol{\alpha}$ represent the vectors composed of z_i, α_i respectively, and based on the n groups of observed values $\{(z_i,\alpha_i), 1 \leq i \leq n\}$, the likelihood function is

$$\begin{aligned} L(\lambda|\boldsymbol{z},\boldsymbol{\alpha}) &= \prod_{i=1}^{n} \{[f(z_i\,|\lambda)\bar{G}_i(z_i)a_i]^{\alpha_i}[g_i(z_i)\bar{F}(z_i\,|\lambda)]^{1-\alpha_i}\} \\ &= A\prod_{i=1}^{n} \{[f(z_i\,|\lambda)]^{\alpha_i}[\bar{F}(z_i\,|\lambda)]^{1-\alpha_i}\} \\ &\propto \prod_{i=1}^{n} \{[f(z_i\,|\lambda)]^{\alpha_i}[\bar{F}(z_i\,|\lambda)]^{1-\alpha_i}\}, \end{aligned}$$

where $A = \prod\limits_{i=1}^{n} [\bar{G}_i(z_i)]^{\alpha_i}[g_i(z_i)]^{1-\alpha_i}$, and A is independent of parameter λ.

3. The Ga–E model under random censoring test

Suppose product life X follows exponential distribution $E(\lambda)$ and the prior distribution of parameter λ is conjugated prior distribution gamma distribution $Ga(\alpha, \beta)$, then this model is called Ga–E model.

Next, we discuss the Ga–E model under random censoring test.

Suppose censored time $Y_i \sim E(\mu), \mu \neq \lambda$, then the likelihood function is

$$L(\lambda\,|\boldsymbol{z}, \boldsymbol{\alpha}) \propto \lambda^{\sum\limits_{i=1}^{n}\alpha_i} e^{-\lambda\sum\limits_{i=1}^{n} z_i}.$$

We know that $(\sum\limits_{i=1}^{n}\alpha_i, \sum\limits_{i=1}^{n} z_i)$ is a sufficient statistic for λ.

Set $\alpha = \sum\limits_{i=1}^{n}\alpha_i, Z = \sum\limits_{i=1}^{n} Z_i$, Let's take the probability distribution of (α, Z).

$$P(\alpha_i = 1) = P(X_i \leq Y_i) = \int_0^\infty \lambda e^{-\lambda x}(\int_x^\infty \mu e^{-\mu y}dy)dx = \lambda(\lambda+\mu)^{-1} \triangleq p,$$

hence, α_i follows 0-1 distribution $b(1, p)$, i.e., $\alpha_i \sim b(1, p)$.

When $\alpha = 0$, $Z \sim Ga(n, \mu)$, and when $\alpha = n$, $Z \sim Ga(n, \lambda)$.

When $\alpha = k\,(1 \leq k < n)$, in $Z_1, Z_2, \cdots, Z_n$ there are k product life, whose sum $W_1 \sim Ga(k, \lambda)$, and there are $n-k$ censored life, whose sum $W_2 \sim Ga(n-k, \mu)$, hence the probability density function (pdf) of $Z = W_1 + W_2$ is

$$\begin{aligned} f_Z(z, k; \lambda) &= \frac{\lambda^k \mu^{n-k}}{\Gamma(k)\Gamma(n-k)} \int_0^z x^{k-1} e^{-\lambda x}(z-x)^{n-k-1} e^{-\mu(z-x)} dx \\ &= \frac{\lambda^k \mu^{n-k}}{\Gamma(k)\Gamma(n-k)} z^{n-1} e^{-\mu z} \int_0^1 t^{k-1}(1-t)^{n-k-1} e^{-z(\lambda-\mu)t} dt \\ &= \left(\frac{\mu^n}{\Gamma(n)} z^{n-1} e^{-\mu z}\right) \lambda^k \mu^{-k} \int_0^1 \left(\frac{\Gamma(n)}{\Gamma(k)\Gamma(n-k)} t^{k-1}(1-t)^{n-k-1}\right) \\ &\quad e^{-z(\lambda-\mu)t} dt \\ &= Ga(z; n, \mu)\lambda^k \mu^{-k} \int_0^1 Be(t; k, n-k) e^{-z(\lambda-\mu)t} dt, \end{aligned}$$

where $Ga(z; n, \mu)$ and $Be(t; k, n-k)$ are the pdfs at z and t of gamma distribution and beta distribution respectively.

To sum up, we can get

$$f_Z(z;k,\lambda)=\begin{cases}Ga(z;n,\mu),\ \ k=0;\\ Ga(z;n,\mu)\lambda^k\mu^{-k}\int_0^1 Be(t;k,n-k)e^{-z(\lambda-\mu)t}dt\ ,\ 1\le k<n;\\ Ga(z;n,\lambda),k=n.\end{cases}$$

The pdf of (α, Z) is

$$\begin{aligned}f_{(\alpha,Z)|\lambda}(k,z)&=P(\alpha=k)f_Z(z,k;\lambda)\\&=C_n^kp^kq^{n-k}f_Z(z,k;\lambda)\\&=b(k;n,p)f_Z(z,k;\lambda).\end{aligned}$$

Since the prior distribution of λ is a gamma distribution $Ga(\alpha,\beta)$, so the joint distribution of α and Z is

$$h((k,z),\lambda)=b(k;n,p)f_Z(z,k;\lambda)Ga(\lambda;\alpha,\beta).$$

Then the marginal distribution of (α, Z) is

$$\begin{aligned}m(k,z)&=\int_0^\infty h((k,z),\lambda)d\lambda\\&=\begin{cases}C_n^k\mu^{n-k}Ga(z;n,\mu)\int_0^\infty\lambda^k(\lambda+\mu)^{-n}Ga(\lambda;\alpha,\beta)d\lambda,\ \ k=0;\\ C_n^k\mu^{n-2k}Ga(z;n,\mu)\int_0^\infty\lambda^{2k}(\lambda+\mu)^{-n}Ga(\lambda;\alpha,\beta)\\ \quad\times\left[\int_0^1 Be(t;k,n-k)e^{-z(\lambda-\mu)t}dt\right]d\lambda\ ,\ 1\le k<n;\\ C_n^k\mu^{n-k}\int_0^\infty\lambda^k(\lambda+\mu)^{-n}Ga(z;n,\lambda)Ga(\lambda;\alpha,\beta)d\lambda,\ \ k=n.\end{cases}\end{aligned}$$

The posterior distribution of λ is

$$\begin{aligned}\pi(\lambda\,|k,z\,) &\propto f_{(\alpha,Z)|\lambda}(k,z)\lambda^{\alpha-1}e^{-\beta\lambda}\\ &\propto \pi_0(\lambda\,|k,z\,)\\ &=\begin{cases}\lambda^{\alpha-1}e^{-\beta\lambda}(\lambda+\mu)^{-n},\ k=0;\\ \lambda^{2k+\alpha-1}e^{-\beta\lambda}(\lambda+\mu)^{-n}\int_0^1 Be(t;k,n-k)e^{-z(\lambda-\mu)t}dt\ ,\ 1\le k<n;\\ \lambda^{2n+\alpha-1}e^{-(z+\beta)\lambda}(\lambda+\mu)^{-n},\ k=n.\end{cases}\end{aligned}$$

Thus

$$E(\lambda\,|k,z)\,=A\int_0^{\infty}\lambda\pi_0(\lambda\,|k,z\,)d\lambda\triangleq e(k,z)=e,$$

where $A^{-1}=\int_0^{\infty}\pi_0(\lambda\,|k,z\,)d\lambda$.

The Monte Carlo method is used to calculate $E(\lambda|k,z)$.

When $k=0$,

$$E(\lambda\,|k,z)\,=\frac{\int_0^{\infty}\lambda^{\alpha}e^{-\beta\lambda}(\lambda+\mu)^{-n}d\lambda}{\int_0^{\infty}\lambda^{\alpha-1}e^{-\beta\lambda}(\lambda+\mu)^{-n}d\lambda}=\frac{E[\lambda(\lambda+\mu)^{-n}]}{E[(\lambda+\mu)^{-n}]},$$

where $\lambda\sim Ga(\alpha,\beta)$, and $E(\lambda|k,z)$ is not a function of z.

Generate $\lambda_i, i=1,2,\cdots,N$ from $Ga(\alpha,\beta)$, hence

$$E[\lambda(\lambda+\mu)^{-n}]\approx\frac{1}{N}\sum_{i=1}^{N}\lambda_i(\lambda_i+\mu)^{-n},\ E[(\lambda+\mu)^{-n}]\approx\frac{1}{N}\sum_{i=1}^{N}(\lambda_i+\mu)^{-n}.$$

When $k=n$,

$$E(\lambda\,|k,z)\,=\frac{\int_0^{\infty}\lambda^{2n+\alpha}e^{-(\beta+z)\lambda}(\lambda+\mu)^{-n}d\lambda}{\int_0^{\infty}\lambda^{2n+\alpha-1}e^{-(\beta+z)\lambda}(\lambda+\mu)^{-n}d\lambda}=\frac{E[\lambda(\lambda+\mu)^{-n}]}{E[(\lambda+\mu)^{-n}]},$$

where $\lambda\sim Ga(2n+\alpha,\beta+z)$.

Generate $\lambda_i, i=1,2,\cdots,N$ from $Ga(2n+\alpha,\beta+z)$, hence

$$E[\lambda(\lambda+\mu)^{-n}]\approx\frac{1}{N}\sum_{i=1}^{N}\lambda_i(\lambda_i+\mu)^{-n},\ E[(\lambda+\mu)^{-n}]\approx\frac{1}{N}\sum_{i=1}^{N}(\lambda_i+\mu)^{-n}.$$

When $1 \leq k < n$,

$$\begin{aligned} E(\lambda\,|k,z) &= \frac{\int_0^\infty \lambda^{2k+\alpha} e^{-\beta\lambda}(\lambda+\mu)^{-n} \int_0^1 Be(t;k,n-k) e^{-z(\lambda-\mu)t} dt d\lambda}{\int_0^\infty \lambda^{2k+\alpha-1} e^{-\beta\lambda}(\lambda+\mu)^{-n} \int_0^1 Be(t;k,n-k) e^{-z(\lambda-\mu)t} dt d\lambda} \\ &= \frac{E[\lambda(\lambda+\mu)^{-n} e^{-z(\lambda-\mu)T}]}{E[(\lambda+\mu)^{-n} e^{-z(\lambda-\mu)T}]}, \end{aligned}$$

where $\lambda \sim Ga(2k+\alpha,\beta), T \sim Be(k,n-k)$, λ and T are independent of each other.

Generate $\lambda_i, i = 1,2,\cdots,N$ from $Ga(2k+\alpha,\beta)$ and $t_i, i = 1,2,\cdots,N$ from $Be(k,n-k)$, hence

$$\begin{aligned} E[(\lambda+\mu)^{-n} e^{-z(\lambda-\mu)T}] &\approx \frac{1}{N}\sum_{i=1}^{N} (\lambda_i+\mu)^{-n} e^{-z(\lambda-\mu)t_i}, \\ E[\lambda(\lambda+\mu)^{-n} e^{-z(\lambda-\mu)T}] &\approx \frac{1}{N}\sum_{i=1}^{N} \lambda_i(\lambda_i+\mu)^{-n} e^{-z(\lambda-\mu)t_i}. \end{aligned}$$

4. The general form of linear decision model on two actions

The so-called linear decision model on two actions refers to that there are only two points a_1 and a_2 in the action space. The state parameter space can be either discrete or continuous probability space. The revenue function is a linear function (or constant) of the state parameter for each action. The general form of its revenue function is

$$Q(\lambda,a) = \begin{cases} m_1\lambda + b_1, & a = a_1, \\ m_2\lambda + b_2, & a = a_2, \end{cases} \quad m_1 > m_2, b_1 < b_2. \tag{1}$$

If $m_1 < m_2$, commutate a_1 and a_2.

To the revenue function Equation (1), let λ_0 denote the equilibrium point, hence

$$m_1\lambda_0 + b_1 = m_2\lambda_0 + b_2 \Rightarrow \lambda_0 = \frac{b_2 - b_1}{m_1 - m_2}.$$

The revenue function is used to calculate the prior mathematical expectations of a_1, a_2 respectively.

$$E_1 = E(m_1\lambda + b_1), E_2 = E(m_2\lambda + b_2).$$

Hence,

$$E_1 - E_2 = (m_1 - m_2)E(\lambda) + (b_1 - b_2) = (m_1 - m_2)[E(\lambda) - \lambda_0].$$

So, according to the prior expectation criterion, when $E(\lambda) > \lambda_0$, a_1 is the optimal action; when $E(\lambda) < \lambda_0$, a_2 is the optimal action; when $E(\lambda) = \lambda_0$, a_1 is equivalent to a_2. According to the criterion of posterior expectation, the optimal action can be selected as follows:

$$\begin{cases} \text{when } E(\lambda\,|k,z) < \lambda_0, a_2 \text{ is the optimal action;} \\ \text{when } E(\lambda\,|k,z) = \lambda_0, a_1 \text{ is equivalent to } a_2; \\ \text{when } E(\lambda\,|k,z) > \lambda_0, a_1 \text{ is the optimal action.} \end{cases}$$

5. EVSI calculation formula

EVSI (Expected Value of Sampling Information) is defined as the increase of the expected income brought by the optimal actions before and after sampling to the decision maker, and its calculation formula for this paper is

$$\text{EVSI} = E_{(\alpha,Z)}\{E_{\lambda|(\alpha,Z)}[Q(\lambda, \delta(\alpha, Z))]\} - E_\lambda[Q(\lambda, a_k)],$$

Where a_k denotes the optimal action before sampling, and $\delta(\alpha, Z)$ denotes the optimal decision function after sampling.

The EVSI of linear decision problem on two actions under the Ga–E model for random censoring test is calculated below.

Let a_1 be the optimal action. Under this assumption, the prior expectation of the revenue function is

$$E(m_1\lambda + b_1) = b_1 + m_1\frac{\alpha}{\beta}.$$

Hence

$$\text{EVSI} = E_{(\alpha,Z)}\{E_{\lambda|(\alpha,Z)}\left[Q(\lambda,\delta(\alpha,Z))\right]\} - \left(b_1 + m_1\frac{\alpha}{\beta}\right).$$

According to the posterior optimal action selection rule, the revenue function is

$$Q(\lambda,\delta(\alpha,Z)) = \begin{cases} Q(\lambda,a_1) = b_1 + m_1\lambda,\ E(\lambda\,|k,z) > \lambda_0; \\ Q(\lambda,a_1) = b_1 + m_1\lambda_0\ E(\lambda\,|k,z) = \lambda_0; \\ Q(\lambda,a_2) = b_2 + m_2\lambda,\ E(\lambda\,|k,z) < \lambda_0. \end{cases}$$

Set $D_1 = \{z\,|e(k,z) > \lambda_0\}, D_2 = \{z\,|e(k,z) < \lambda_0\}$, thus

$$\begin{aligned} & E_{(\alpha,Z)}\{E_{\lambda|(\alpha,Z)}\left[Q(\lambda,\delta(\alpha,Z))\right]\} \\ = & \sum_{k=0}^{n}\left[\int\limits_{D_1}(b_1 + m_1 e)m(k,z)dz + \int\limits_{D_2}(b_2 + m_2 e)m(k,z)dz\right] \\ = & \sum_{k=0}^{n}\left[\int_0^{\infty}(b_1 + m_1 e)m(k,z)dz + \int\limits_{D_2}(b_3 + m_3 e)m(k,z)dz\right], \end{aligned}$$

where $b_3 = b_2 - b_1, m_3 = m_2 - m_1$.

Hence

$$\text{EVSI} = \sum_{k=0}^{n}\left[\int_0^{\infty}(b_1 + m_1 e)m(k,z)dz + \int\limits_{D_2}(b_3 + m_3 e)m(k,z)dz\right] - \left(b_1 + m_1\frac{\alpha}{\beta}\right).$$

Solving D_1, D_2 can be accomplished by drawing e's plan about z with the help of computer software. First k is fixed, then set $h(z) = E(\lambda\,|k,z)$. Draw the plane graph of $y = h(x) - \lambda_0$ through computer software, then $D_1 = \{x\,|y > 0\}, D_2 = \{x\,|y < 0\}$.

The specific steps to solve $E_{(\alpha,Z)}\{E_{\lambda|(\alpha,Z)}\left[Q(\lambda,\delta(\alpha,Z))\right]\}$ are given below.

Set

$$r(k) = \int\limits_{D_1} (b_1 + m_1 e) m(k, z) dz + \int\limits_{D_2} (b_2 + m_2 e) m(k, z) dz.$$

(1) When $k = 0$, $e = c$ is a constant function of z, and judge the magnitude of $h(z)$ and λ_0.

When $e > \lambda_0$, then $D_1 = (0, +\infty), D_2 = \varnothing$, hence

$$r(k) = (b_1 + m_1 e) \int_0^\infty m(0, z) dz.$$

When $e < \lambda_0$, then $D_1 = \varnothing, D_2 = (0, +\infty)$, hence

$$r(k) = (b_2 + m_2 e) \int_0^\infty m(0, z) dz.$$

Meanwhile,

$$\begin{aligned}
\int_0^\infty m(0, z) dz &= \mu^n \int_0^\infty Ga(z; n, \mu) dz \int_0^\infty (\lambda + \mu)^{-n} Ga(\lambda; \alpha, \beta) d\lambda \\
&= \mu^n \int_0^\infty (\lambda + \mu)^{-n} Ga(\lambda; \alpha, \beta) d\lambda \\
&= \mu^n E[(\lambda + \mu)^{-n}],\ \lambda \sim Ga(\alpha, \beta).
\end{aligned}$$

(2) When $k = n$, then set $D_2 = [a, b)$, hence

$$\begin{aligned}
&\int_0^\infty (b_1 + m_1 e) m(k, z) dz \\
&= \int_0^\infty (b_1 + m_1 e) dz \int_0^\infty \lambda^n (\lambda + \mu)^{-n} Ga(z; n, \lambda) Ga(\lambda; \alpha, \beta) d\lambda \\
&= E\{[b_1 + m_1 e(n, Z)] \lambda^n (\lambda + \mu)^{-n}\}, \lambda \sim Ga(\alpha, \beta), Z \sim Ga(n, \lambda), \\
&\approx \frac{1}{N} \sum_{i=1}^{N} [b_1 + m_1 e(n, z_i)] \lambda_i^n (\lambda_i + \mu)^{-n}, \lambda_i \sim Ga(\alpha, \beta), z_i \sim Ga(n, \lambda_i).
\end{aligned}$$

Meanwhile,

$$\begin{aligned}
&\int_a^b (b_3+m_3e)m(k,z)dz\\
&=\int_a^b (b_3+m_3e)dz\int_0^\infty \lambda^n(\lambda+\mu)^{-n}[F(b;n,\lambda)-F(a;n,\lambda)]\\
&\qquad\qquad\frac{Ga(z;n,\lambda)}{[F(b;n,\lambda)-F(a;n,\lambda)]}Ga(\lambda;\alpha,\beta)d\lambda\\
&=E\{[b_3+m_3e(n,Z)][F(b;n,\lambda)-F(a;n,\lambda)]\lambda^n(\lambda+\mu)^{-n}\}\\
&\approx\frac{1}{N}\sum_{i=1}^N[b_3+m_3e(n,z_i)][F(b;n,\lambda_i)-F(a;n,\lambda_i)]\lambda_i^n(\lambda_i+\mu)^{-n},
\end{aligned}$$

where $F(a;n,\lambda)$ is the distribution function at a of $Ga(n,\lambda)$, $\lambda\sim Ga(\alpha,\beta), Z\sim TGa(n,\lambda,a,b), \lambda_i\sim Ga(\alpha,\beta), z_i\sim TGa(n,\lambda_i,a,b)$, and $TGa(n,\lambda,a,b)$ is the gamma distribution $Ga(n,\lambda)$ on the interval $[a,b)$.

(3) When $1\le k<n$, then set $D_2=[a,b)$, hence

$$\begin{aligned}
&\int_0^\infty (b_1+m_1e)m(k,z)dz\\
&=C_n^k\mu^{n-2k}\int_0^\infty (b_1+m_1e)Ga(z;n,\mu)dz\\
&\qquad\int_0^\infty \lambda^{2k}(\lambda+\mu)^{-n}Ga(\lambda;\alpha,\beta)d\lambda\int_0^1 Be(t;k,n-k)e^{-z(\lambda-\mu)t}dt\\
&=C_n^k\mu^{n-2k}E\{[b_1+m_1e(k,z)]\lambda^{2k}(\lambda+\mu)^{-n}e^{-Z(\lambda-\mu)T}\}\\
&\approx C_n^k\mu^{n-2k}\frac{1}{N}\sum_{i=1}^N[b_1+m_1e(k,z_i)]\lambda_i^{2k}(\lambda_i+\mu)^{-n}e^{-z_i(\lambda_i-\mu)t_i},
\end{aligned}$$

where

$$\lambda\sim Ga(\alpha,\beta), T\sim Be(k,n-k), Z\sim Ga(n,\mu);$$

$$\lambda_i\sim Ga(\alpha,\beta), t_i\sim Be(k,n-k), z_i\sim Ga(n,\mu).$$

Meanwhile,

$$\begin{aligned}
&\int_a^b (b_3 + m_3 e) m(k,z) dz \\
&= C_n^k \mu^{n-2k} \int_a^b (b_3 + m_3 e) Ga(z; n, \mu) dz \\
&\quad \int_0^\infty \lambda^{2k} (\lambda + \mu)^{-n} Ga(\lambda; \alpha, \beta) d\lambda \int_0^1 Be(t; k, n-k) e^{-z(\lambda-\mu)t} dt \\
&= BC_n^k \mu^{n-2k} E\{[b_3 + m_3 e(k,z)] \lambda^{2k} (\lambda + \mu)^{-n} e^{-Z(\lambda-\mu)T}\} \\
&\approx BC_n^k \mu^{n-2k} \frac{1}{N} \sum_{i=1}^N [b_3 + m_3 e(k, z_i)] \lambda_i^{2k} (\lambda_i + \mu)^{-n} e^{-z_i(\lambda_i - \mu)t_i},
\end{aligned}$$

where

$$B = F(b; n, \mu) - F(a; n, \mu);$$

$$\lambda \sim Ga(\alpha, \beta), T \sim Be(k, n-k), Z \sim TGa(n, \mu, a, b);$$

$$\lambda_i \sim Ga(\alpha, \beta), t_i \sim Be(k, n-k), z_i \sim TGa(n, \mu, a, b).$$

6. Conclusions

In this paper, we give the likelihood function based on the observed data under the random censoring test, derive the posterior distribution of parameter of exponential distribution life for the Ga–E model under random censoring test, and focuses on the EVSI calculation formula of linear decision problem on two actions under the Ga–E model for random censoring test. Most of the calculations are done using the Monte Carlo method.

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