Analysis of Flow of an Incompressible MHD Third Grade Fluid In An Inclined Rotating Cylindrical Pipes With Isothermal Wall and Joule Heating

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Abstract

In this paper, we considered the analysis of flow of an incompressible MHD third grade fluid in an inclined rotating cylindrical pipes with isothermal wall and Joule heating. The governing equations of the flow field are solved using perturbation method. Results show that the Eckert and the Grashhof parameters reduce the velocity of the fluid flow, while the magnetic field parameter, the Grashhof number, the Eckert number and the third grade parameter increase the temperature of the system. . It is further discovered that the temperature profiles converges to zero at the point where $R = 2$, which shows that isothermal nature of the outer cylinder.

Keywords: *Incompressible MHD, isothermal wall, Joule heating.*

1. Introduction

In the study of the flow of an incompressible magnetohydrodynamic third grade fluid in a rotating cylindrical annulus with isothermal wall and Joule heating, many investigators have considered the flow of third grade fluid in different forms. Some of the foremost works are Fosdick and Rajagopal (1980), who examined the thermodynamics and stability of fluids of third grade and showed restrictions on the stress constitutive equation was concern with the relation between thermodynamics and stability for a class of non-Newtonian incompressible fluids of the differential type, and further introduced the additional thermodynamical restriction that the Helmholtz free energy density be at a minimum value when the fluid is locally at rest. They gave detailed attention to the special case of fluids of grade 3 and arrived at fundamental inequalities which restricts its temperature dependent. They discovered that these inequalities requires that a body of such a fluid be stable in the sense that its total kinetic energy must tend to zero in time, no matter what its previous mechanical and thermal fields, provided it is both mechanically isolated and immersed in a thermally passive environment at constant temperature from some finite time onward.

Rajagopal *et al* (1986) examined the flow past an infinite porous flat plates with suction. They discovered that the non-Newtonian fluid mechanics affords an excellent opportunity for studying many of the mathematical methods which have been developed to analyse non-linear problems in mechanics. They established an existence theorem using the shooting and investigated the problem using perturbation analysis and numerical method and concluded that since perturbation method did not converge suitably, it was not the appropriate tool for the problem. Obi *et al* (2017) examined annular flow of an incompressible MHD third grade fluid in a rotating concentric cylinders with isothermal walls and Joule heating.
Okoya (2011) considered the disappearance of criticality for reactive third grade fluid with model viscosity in a flat channel. Mahmood and Nargis (2012) on thin film flow of a third grade fluid through porous medium over an inclined plane, employed the perturbation and homotopy perturbation methods and the results obtained from the two methods are in close agreement. Masoudi and Christie (1995) analysed the effect of temperature-dependent viscosity for the three separate cases treated by Szeri and Rajagopal (1998).

Aiyesimi *et al* (2014) dealt with the effects of magnetic field on the MHD flow of third grade fluid through inclined channel with Ohmic heating. They analysed the Couette flow, Poiseuille and Couette-Poiseuille flow and solved the resulting non-linear differential equation by employing regular perturbation technique.

Makinde (2005) examined the hydrodynamic ally and thermally developed Reynolds viscosity liquid film along an inclined heated plate: an exploitation of Hermite-pade approximation technique. Ayub, et al (2003) examined the exact flow of third grade fluid past a porous plate using homotopy analysis method. Yurusoy and Pakdemirli (2002) considered approximate analytical solutions for the flow of a third grade fluid in a pipe. Makinde (2007)

studied thermal stability of a reactive third grade fluid in a cylindrical pipe: An exploitation of Hermite Pade approximation technique.

Baldoni *et al* (1993) studied the helical flow of a third grade fluid between eccentric cylinders, using a domain perturbation approach. They discovered a secondary flow contrary to the initial analysis. The consistency of the slow flow approximation has been tested considering the behaviour of the fluid at intermediate and high Reynolds number.

According to Rajagopal and Mollica (1999) on secondary deformation due to axial shearing of annular region between two eccentrically placed cylinders followed the works of Fosdick and kao (1978) and extended a conjecture of Ericksen's (1956) for non linear fluid, to nonlinear elastic solids and showed that unless the material modulli of an isotropic elastic material satisfied certain special relations, axial shearing of cylinders would be necessarily accompanied by secondary deformation if the cross-section were not a circle or the annular region between concentric circles.

2. Mathematical Formulation:

Following Feiz-Dizaji et al (2007), we consider the steady flow of an incompressible MHD third grade fluid in a rotating cylindrical annulus with isothermal wall. Pressure-gradient was assumed to have induced the fluid motion. For magnetohydrodynamically developed flow, both the velocity and the temperature fields depend on r only. Considering [5, 10, 14] and neglecting the reacting viscous fluid assumption, the governing equations for the momentum and energy balance can be represented as:

$$\mu\left(\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2}\right)+\beta\left(\frac{du}{dr}-\frac{u}{r}\right)^2\left(6\frac{d^2u}{dr^2}-\frac{2}{r}\frac{du}{dr}+2\frac{u}{r^2}\right)+\rho g\beta\left(T-T_0\right)\sin\phi-\sigma B_0^2 u=0 \quad (1)$$

$$k\left(\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}-\frac{T}{r^2}\right)+\beta\left(\frac{du}{dr}-\frac{u}{r}\right)^2\left(\delta+\beta_3\left(\frac{du}{dr}-\frac{u}{r}\right)^2\right)+\sigma B_0^2 u^2=0 \quad (2)$$

subject to the conditions

$$\frac{dT}{dr}\left(r_1\right)=0,\ T\left(r_2\right)=T_0\ u=r_1\omega_1\ at\ r=r_1;\ u=r_2\omega_2\ at\ r=r_2 \quad (3)$$

We introduce the following non-dimensional variables into equations for non-dimensionalisation

$$\theta=\frac{T}{T_0},\bar{r}=\frac{r}{a},\bar{u}=\frac{u}{u_0}\ \sigma=\frac{\beta_3 u^2}{a^2\delta},\ r=r_1\bar{r}\ and\ u=r_1\omega\bar{u} \quad (4)$$

where

$r_1<r_2$ and r_1 is the radius of the inner cylinder, r_2 is the radius of the outer cylinder, ω_1 is the angular velocity of the inner cylinder , ω_2 is the angular velocity of the outer cylinder T is the temperature of the cylinder, T_0 is the plate temperature, μ is the coefficient of dynamic viscosity of the fluid, u is the fluid velocity, u_0 is the characteristic fluid velocity, B_0 is the magnetic field effect, β_3 is the material coefficient and σ is the heat reaction.

Then substituting equation $\left(4\right)$ into equations $\left(1\right)\ to\left(3\right)$, we obtain

$$\left(\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2}\right)+\mu\left(\frac{du}{dr}-\frac{u}{r}\right)^2\left(6\frac{d^2u}{dr^2}-\frac{2}{\bar{r}}\frac{du}{dr}+\frac{2u}{r^2}\right)+\delta\theta-\tau u=0 \quad (5)$$

$$\left(\frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}-\frac{\theta}{r^2}\right)+\left(\frac{du}{dr}-\frac{u}{r}\right)^2+B_r\beta\left(\frac{du}{dr}-\frac{u}{r}\right)^4+B_r\tau u^2=0 \quad (6)$$

3. METHOD OF SOLUTION

In order to solve equation (5), we introduce perturbation series as follows:

$$u = u_0 + \beta u_1 + \beta u_2 + 0(\beta)^3, \theta = \theta_0 + \beta \theta_1 + \beta \theta_2 + 0(\beta)^3 \ and \ \tau = \beta M \qquad (7)$$

Substituting equation (7) into equation (5), we obtain order β^0, β and β^2 problems as follows:

$$\beta^0 : \frac{d^2 u_0}{dr^2} + \frac{1}{r}\frac{du_0}{dr} - \frac{u_0}{r^2} = 0 \qquad (8)$$

$$\beta : \frac{d^2 u_1}{dr^2} + \frac{1}{r}\frac{du_1}{dr} - \frac{u_1}{r^2} + \gamma \frac{du_0}{dr}\frac{d^2 u_0}{dr^2} + G\theta_0 - Mu_0 = 0 \qquad (9)$$

$$\beta^2 : \left(\left(\frac{d^2 u_2}{dr^2} + \frac{1}{r}\frac{du_2}{dr} - \frac{u_2}{r}\right) - \frac{12u_0}{r}\left(\frac{du_1}{dr}\right)\left(\frac{d}{dr}\left(\frac{du_0}{dr}\right)\right) - \frac{12u_0}{r}\left(\frac{du_1}{dr}\right)\left(\frac{d}{dr}\left(\frac{du_0}{dr}\right)\right) - \frac{12u_0}{r}\left(\frac{du_0}{dr}\right)\left(\frac{d}{dr}\left(\frac{du_1}{dr}\right)\right)\right.$$

$$+ \frac{6u_0^2}{r^2}\left(\frac{d}{dr}\left(\frac{du_1}{dr}\right)\right) - \frac{12u_0 u_1}{r^3}\left(\frac{du_0}{dr}\right) + \frac{6u_1}{r^2}\left(\frac{du_0}{dr}\right)^2 + \frac{12u_0}{r^2}\left(\frac{du_0}{dr}\right)\left(\frac{du_1}{dr}\right) - \frac{6}{r}\left(\frac{du_0}{dr}\right)^2\left(\frac{du_1}{dr}\right)$$

$$+ 6\left(\frac{du_0}{dr}\right)^2\left(\frac{d}{dr}\left(\frac{du_1}{dr}\right)\right) + \frac{6u_0^2 u_1}{r^4} + \frac{12u_0 u_1}{r^2}\left(\frac{d}{dr}\left(\frac{du_0}{dr}\right)\right) - \frac{6u_0^2}{r^3}\left(\frac{du_1}{dr}\right) + 12\left(\frac{du_0}{dr}\right)\left(\frac{du_1}{dr}\right)\left(\frac{d}{dr}\left(\frac{du_0}{dr}\right)\right)$$

$$\left. - \frac{12u_1}{r}\left(\frac{du_0}{dr}\right)\left(\frac{d}{dr}\left(\frac{du_0}{dr}\right)\right)\right) + G\theta_1 - Mu_1 = 0 \qquad (10)$$

Solving equations (8), (9) and (10), using the condition (3), we obtain

$$u_0(r) = \frac{R(R-\omega)}{(-1+R^2)r} + \frac{(\omega R-1)r}{-1+R^2} \qquad (11)$$

that is

$$u_0(r) = \frac{A}{r} + Br \qquad (12)$$

where

$$A = \frac{R(R-\omega)}{(-1+R^2)} \ and \ B = \frac{(\omega R-1)}{-1+R^2} \qquad (13)$$

$$u_1(r) = r\left(-\frac{721}{4374} - \frac{2}{3}GA\ln(2) - \frac{127}{1080}M - \frac{5}{8}GB - \frac{10}{27}M\ln(2) - \frac{80}{81}\ln(2) + \frac{1}{4}GA\right)$$

$$+\frac{1}{r}\left(\frac{2611}{1458} + \frac{2}{3}GA\ln(2) + \frac{49}{270}M + \frac{1}{2}GB + \frac{10}{27}M\ln(2) + \frac{80}{81}\ln(2)\right) + \frac{1}{87480}\frac{1}{r^5}\left(-72000r^3\right.$$

$$+24300M\ln(r)r^6 + 43740GA(r)r^6 - 21870GAr^6 + 8505Mr^8 - 40000 - 12150Mr^6$$

$$\left.-1944Mr^6 + 10935GBr^8 + 2160r^9 + 64800\ln(r)r^6 - 32400r^6\right) \tag{14}$$

$$u_2(r) = \frac{1}{661348800}\frac{1}{r^9}\left(653184000M\ln(2)\ln(r)r^{10} + 440899200GB\ln(r)r^{10} - 3124396800GA\ln(2)r^{10}\right.$$

$$+122472000M^2\ln(2)\ln(r)r^{10} - 1306368000GA\ln(2)r^7 - 146994000GAr^9 + 41334300MGAr^{10}$$

$$+103333575MGBr^{10} - 3509184000r^7 - 10124587200r^8 + 2880000000r^3 + 165337200MGB\ln(r)r^{10}$$

$$+587865600GA\ln(2)\ln(r)r^{10} + 41334300MGAr^{12}\ln(r) - 55112400MGA\ln(2)r^{12}$$

$$-2069776800GAr^8 - 110224800G\theta_1 r^8 + 330674400r^{10}G\theta_1 - 220449600G\theta_1 r^{11}$$

$$+30618000M^2\ln(2)r^8 + 3147984000M\ln(2)r^8 - 29243220000r^4 + 612360000GAr^6$$

$$+30240000000r^6 + 30248800000Mr^6 + 3717025200GBr^8 + 1034208000Mr^9 + 341105580Mr^8$$

$$+3628800000r^9 - 21432600r^{14}M + 6345216Mr^{15} - 571536000Mr^{11} - 53430300Mr^{12}$$

$$-38425590M^2 r^{12} + 28796229M^2 r^{10} + 2679075r^{14}M^2 - 385685496Mr^{10} - 419904r^{15}M^2$$

$$-296352000r^4M - 816480000GBr^4 - 1935360000\ln(2)r^7 + 1579132800\ln(r)r^{10}$$

$$-4628736000\ln(2)r^{10} - 81648000M\ln(2)r^{12} + 22963500M^2 r^{12}\ln(r) - 725760000M\ln(2)r^7$$

$$+37889775MGBr^8 - 10333575MGAr^8 + 5519404800GA\ln(2)r^8 + 1344000000$$

$$-1735776000M\ln(2)r^{10} + 8709120000\ln(2)\ln(r)r^{10} + 60011280M^2\ln(r)r^{10}$$

$$+752204880M\ln(r)r^{10} - 30618000M^2\ln(2)r^{12} + 612360000Mr^{12}\ln(r) + 7370190M^2 r^8$$

$$+8176896000\ln(2)r^8 + 3444525MGBr^{14} - 51667875MGBr^{12} - 31000725MGAr^{12}$$

$$-6531840r^{15} + 5959983040r^{10} + 97977600GAr^{10} + 220449600MGA\ln(2)\ln(r)r^{10}$$

$$-1158381000GBr^{10} - 27556200GBr^{14} - 110224800GAr^{12} - 734832000GBr^{11}$$

$$-1088640000r^4GA\ln(2) - 979776000GBr^7 - 604800000r^4M\ln(2) - 355622400Mr^7$$

$$-1612800000r^4\ln(2) + 55112400MGA\ln(2)r^8 - 272160000r^{12}) \tag{15}$$

4.0 Heat Transfer Analysis

Usng the series (8) in (6) and simplifying, we obtain

$$\beta^0 : \frac{d^2\theta_0}{dr^2} + \frac{1}{r}\frac{d\theta_0}{dr} - \frac{\theta_0}{r^2} = 0 \tag{16}$$

$$\beta : \frac{d^2\theta_1}{dr^2} + \frac{1}{r}\frac{d\theta_1}{dr} - \frac{\theta_1}{r^2} + 6B_r\frac{u_0^2}{r^2}\left(\frac{du_0}{dr}\right)^2 - 4B_r\frac{u_0^3}{r^3}\left(\frac{du_0}{dr}\right) + B_r\left(\frac{du_0}{dr}\right)^4 - 6B_r\frac{u_0}{r}\left(\frac{du_0}{dr}\right)^3$$

$$+\left(\frac{du_0}{dr}\right)^2 - \frac{u_0}{r}\left(\frac{du_0}{dr}\right) + \frac{u_0^2}{r^2} + B_r\frac{u_0^4}{r^4} + Mu_0^2 = 0 \tag{17}$$

$$\beta^2 : \frac{d^2\theta_2}{dr^2} + \frac{1}{r}\frac{d\theta_2}{dr} - \frac{\theta_2}{r^2} - \frac{12B_ru_0}{r}\left(\frac{du_0}{dr}\right)^2\left(\frac{du_0}{dr}\right) + 2E_c\left(\frac{du_0}{dr}\right)\left(\frac{du_1}{dr}\right) - \frac{4B_ru_1}{r}\left(\frac{du_0}{dr}\right)^3$$

$$+\frac{4B_ru_0^3u_1}{r^4} + \frac{12B_ru_0u_1}{r^2}\left(\frac{du_0}{dr}\right)^2 - \frac{2E_cu_1}{r}\left(\frac{du_0}{dr}\right) - \frac{12B_ru_0^2}{r^3}\left(\frac{du_0}{dr}\right) + \frac{12B_ru_0^2}{r^2}\left(\frac{du_0}{dr}\right)\left(\frac{du_1}{dr}\right)$$

$$-\frac{12B_ru_0^3}{r^3}\left(\frac{du_1}{dr}\right) - \frac{2B_cu_0}{r}\left(\frac{du_1}{dr}\right) + 4B_r\left(\frac{du_0}{dr}\right)^3\left(\frac{du_1}{dr}\right) + \frac{2E_cu_0u_1}{r^2} + 2B_rMu_0u_1 = 0 \tag{18}$$

$$\frac{d\theta}{dr}(1) = 0, \ \theta(R) = 1 \tag{19}$$

Solving equations (16), (17) and (18), using the condition (19) we obtain

$$\theta_0(r) = -\frac{R}{(-1+R^2)r} + \frac{Rr}{-1+R^2} \tag{20}$$

that is

$$\theta_0(r) = \frac{E}{r} + Fr \tag{21}$$

where

$$E = \frac{R}{\left(-1+R^2\right)} \ and \ F = \frac{Rr}{1+R^2} \tag{22}$$

$$\theta_1(r) = r\left(\frac{8}{27}E_c\ln(2) - \frac{962867}{2296350}B_r + \frac{15056}{42525}B_rM - \frac{367}{2025}E_c\right) + \frac{1}{r} + \frac{8}{27}E_c\ln(2) + \frac{1065523}{2296350}B_r$$

$$-\frac{6614}{42525}B_rM + \frac{2594}{6075}E_c - \frac{1}{918540}\frac{1}{r^6}\left(-756000B_rr^6 - 283500B_rMr^6 + 2916B_rMr^{12} + 324B_rr^{12}\right.$$

$$-19845B_rMr^{11} + 6804E_cr^{10} + 37044r^{10}B_rM - 42525B_rMr^9 + 18900B_rr^9 + 264600B_rMr^8$$

$$+378000E_cr^4 + 340200E_cr^7\ln(r)170100E_cr^7 + 40000B_r + 210000B_rr^3) \tag{23}$$

$$\theta_2(r) = -\frac{1}{589161780800}\frac{1}{r^{10}}\left(-359251200000B_rr^{12} + 2664766781600B_rr^9 + 19399564880000E_cr^{10}\right.$$

$$+29785909923B_rM^2r^9 - 61585920B_rMr^{18} + 1563341472000E_cr^8 - 3126682944000B_rr^{10}$$

$$-84873096000E_cMr^{12} - 3182741100E_cr^{15}M - 8911675080B_rM^2r^{13} - 81356924880B_rM^2r^{12}$$

$$-1220050755B_rM^2r^{17} - 20482707168B_rM^2r^{14} + 2546192880B_rM^2r^{16} + 5486439420B_rM^2r^{15}$$

$$+117879300B_rMr^{17} + 1392322932E_cMr^{11} + 192836581740B_rM^2r^{11} + 138568320B_rM^2r^{18}$$

$$+748268928E_cr^{16}M - 2728063800E_cMr^{13} + 15900607800B_rMr^{15} - 83977213320B_rMr^{13}$$

$$-68163923520 B_r M r^{14} + 1993787136000 B_r \ln(2) r^{11} + 21555072000 B_r r^{13} \ln(2)$$

$$+237105792000 E_c \ln(2) r^{11} + 703503662400 E_c \ln(r) r^{11} - 1724405760000 B_r \ln(2) r^{10}$$

$$+862202880000 E_c \ln(2) r^{8} + 182476800000 B_r \ln(2) r^{4} + 718502400000 B_r \ln(2) r^{7}$$

$$+132024816000 B_r M r^{7} - 351920230200 E_c G A r^{9} - 619270482600 E_c G B r^{9} - 187229750400 B_r M \ln(2) r^{9}$$
$$+183007613250 B_r M G A r^{11} - 872980416000 B_r G B r^{10} + 363741840000 B_r G B r^{7} - 269438400000 B_r G B r^{8}$$

$$+556794453600 B_r G B r^{9} + 92378880000 B_r G B r^{4} + 654735312000 E_c G A r^{15} + 436490208000 E_c G B r^{8}$$

$$-107775360000 B_r G A r^{6} + 1345806316800 B_r G A \ln(2) r^{11} + 14549673600 B_r r^{13} G A \ln(2)$$

$$+97401981600 B_r M^{2} \ln(2) r^{9} + 476007840000 B_r M \ln(r) r^{9} - 412240752000 E_c M \ln(2) r^{9}$$

$$-742033353600 E_c G A \ln(2) r^{9} + 727483680000 B_r G A \ln(r) r^{9} + 39083536800 r^{13} B_r$$

$$-831409920 E_c r^{16} - 102643200 B_r r^{18} - 1235636376480 E_c r^{11} + 374500756800 B_r r^{11}$$

$$-48498912000 E_c r^{13} - 5388768000 B_r r^{15} - 24552574200 B_r M G B r^{13} + 10230239250 B_r M G B r^{15}$$

$$+3273676560 B_r M r^{16} G B + 16368382800 B_r M G B r^{12} - 32736765600 B_r M G B r^{14} + 232021826190 B_r M G B r$$

$$-660124080000 B_r M \ln(r) r^{11} + 387991296000 E_c \ln(2) \ln(r) r^{11} + 71293400640 E_c M \ln(r) r^{11}$$

$$+4526561200 B_r r^{14} M \ln(r) + 86220288000 B_r r^{12} M \ln(2) - 60354201600 B_r r^{14} M \ln(2)$$

$$-40415760000 B_r r^{13} M \ln(2) - 18187092000 B_r M^{2} \ln(2) r^{13} + 32332608000 B_r M^{2} \ln(2) r^{12}$$

$$+16974619200 B_r M^{2} r^{14} \ln(r) - 22632825600 B_r M^{2} r^{14} \ln(2) - 12124728000 B_r M r^{15} \ln(r)$$

$$+3410079750 B_r M G A r^{15} - 145496736000 B_r M G A r^{12} - 1023023925 B_r M r^{17} G B - 16295634432 B_r M r^{14} G A$$

$$-363741840000 B_r G B \ln(r) r^{11} + 196420593600 E_c G B \ln(r) r^{11} + 145496736000 E_c M \ln(2) \ln(r) r^{11}$$

$$+160046409600 E_c G A \ln(2) r^{11} - 1163973888000 B_r G A \ln(2) r^{10} + 484989120000 B_r B A \ln(2) r^{7}$$

$$+123171840000 B_r G A \ln(2) r^{4} + 581986944000 E_c G A \ln(2) r^{8} + 123785894925 B_r M G B r^{9}$$

$$-24625322568B_rMGAr^9-804543062400B_rGA\ln(2)r^9+109104053080B_rMr^9+306421315200B_rMr^{12}$$

$$+1302784560000B_rr^7-798336000000B_rr^6-332640000000B_rr^3-538876800000E_cr^7$$

$$-102643200000E_cr^4-349272000000B_rMr^8+491638724280B_rMr^{11}-1489365662400B_rMr^{10}$$

$$-79833600000B_rMr^6+68428800000B_rM\ln(2)r^4-404309159100B_rGBr^9-1576811253600E_cr^9$$

$$-89600000000B_r+33530112000B_rMr^4-32736765600B_rMGA\ln(2)r^{13}$$

$$+58198694400B_rMGA\ln(2)r^{12}+30554314560B_rMr^{14}GA\ln(r)-40739086080B_rMr^{14}GA\ln(2)$$

$$-8184191400B_rMGAr^{15}\ln(r)+10912255200B_rMr^{15}GA\ln(2)+109122552000B_rMGAr^{12}\ln(r)$$

$$+1094699350800B_rGBMr^{11}-301703648400B_rGAr^{11}+10912255200B_rr^{13}GB$$

$$-278262507600E_cGAr^{11}+295994922300E_cGBr^{11}-145496736000B_rr^{14}GAr^{12}$$

$$-109122552000E_cGBr^{12}-454677300B_rr^{17}GB-14549673600B_rr^{14}GB-24552574200E_cGAr^{13}$$

$$-4092095700E_cGBr^{15}-1818709200B_rr^{15}GA+363741840000E_cMr^{10}-118822334400B_rM^2r^{10}$$

$$+158429779200E_cMr^8+26943840000B_rM\ln(2)r^2+323326080000E_cM\ln(2)$$

$$-242494560000B_rM^2\ln(2)r^{10}-1293304320000B_rM\ln(2)r^{10}$$

$$-4546773000B_rM^2r^{15}\ln(r)+161663040000B_rMr^{12}\ln(r)+6062364000B_rM^2r^{15}\ln(2)$$

$$+60623640000B_rM^2r^{12}\ln(r)+161663040000B_rM\ln(2)r^{15}+88914672000E_cM\ln(2)r^{11}$$

$$+1141050240000B_rM\ln(2)r^{11}+147517524000B_rM^2\ln(2)r^{11}-327367656000B_rMGBr^{10}$$

$$-436490208000B_rMGA\ln(2)r^{10}+261894124800E_cGA\ln(2)\ln(r)r^{11}$$

$$+265531543200B_rMGA\ln(2)r^{11}+330865920000B_rr^4-433528310160E_cMr^9$$

$$-1191915648000B_r\ln(2)r^9-1099308672000E_c\ln(2)r^9+2395008000000B_r\ln(r)r^9$$

$$+175323566880B_rMGA\ln(2)r^9)\hspace{3cm}(24)$$

5.0 Results and Discussion

The momentum $u(r)$ and temperature $\theta(r)$ profiles were examined with various values of the physical parameters. In figure 1, the Eckert number was varied between 0.5 and 4.5, while the other parameters were kept constant that is when $p = -1, G = 1, M = 0.5, \beta = 0.1 \text{ and } B_r = 1$. Results show that a little increase in the Eckert number slightly reduced the velocity of the fluid flow. Results further show that the velocity profiles converges to 0.5 at the point $R = 2$ satisfying the boundary condition. Figure 2 shows the Grashhof number was varied between 1 and 9 while the other parameters were held constant, that is $p = -1, G = 1, M = 0.5, \beta = 0.1 \text{ and } B_r = 1$. It is discovered that the velocity profiles converged normally to 0.5 at the point when $R = 1$. Results further show that the Grashhof parameter has the tendency of slowing down the velocity of the fluid flow in the annulus of the concentric rotating cylinders.

In figure 3, the magnetic field parameter was varied between 0.5 and 2.5, with the other parameters kept constant, that is $\beta = 0.1, E_c = 0.5, G = 1 \text{ and } B_r = 1$. Results revealed that increase in the magnetic field increases the temperature of the system . It is further discovered that the temperature profiles converges to zero at the point where   , which shows that the wall of the outer cylinder is isothermally stable. This is due to the effects of rotation of the cylinders.

$$R = 2$$

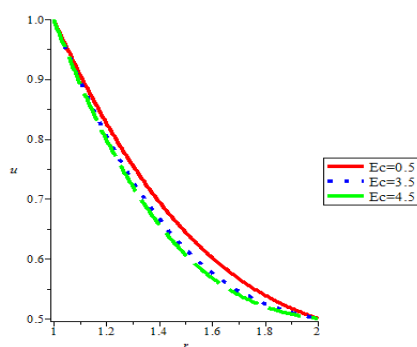Figure 1: Velocity Profile For
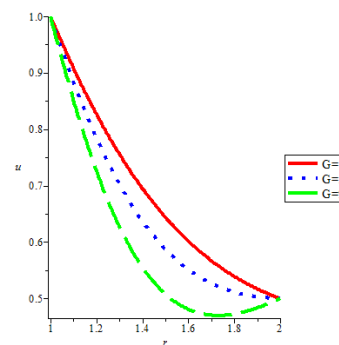Various Values Of The Eckert Number

Figure 2 Velocity Profile For Various
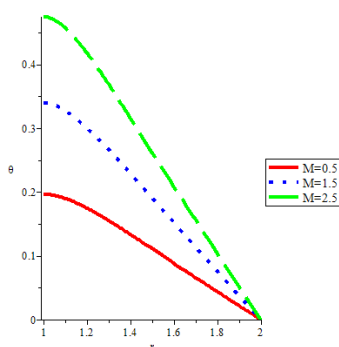Values Of The Grashhof Number

Figure 3: Temperature Profile For
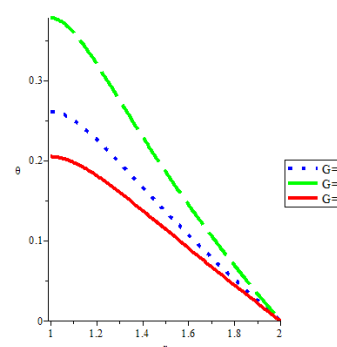Various Values Of The Magnetic Parameter

Figure 4: Temperature Profile For
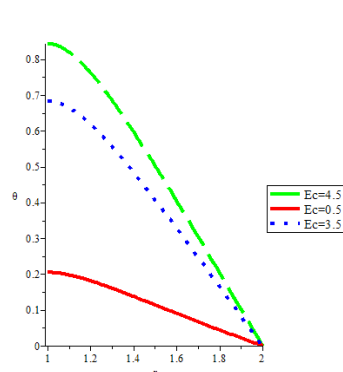Various Values Of The Grashhof Number

Figure 5: Temperature Profile For
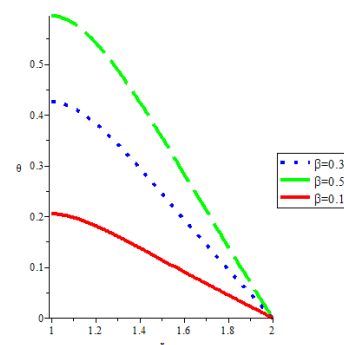Various Values Of The Eckert Number

Figure 6: Temperature Profile For
Various Values Of The Third Grade Parameters

In figure 4, the Grashhof number was varied between 1 and 9 and other parameters were kept constant that is β =0.1, E_c =0.5, M=0.5 and B_r =1. Results show that within the inclined rotating cylindrical annulus, that increase in Grashof number increases the temperature as the profiles converges to zero that is when $R = 2$, confirming the boundary condition, which means that the wall temperature is isothermally stable   In figure 5, the Eckert number was varied between 0.5 and 4.5 in the inclined rotating cylindrical annulus when the other parameters M=0.5, β =0.1, G=1 and B_r =1 were kept constant. Results show that temperature increases with increase in the Eckert number, leading to the convergence of the profiles at zero when $R = 2$.

Figure 6 shows the third grade parameter was varied between 0.1 and 0.5 in the inclined rotating cylindrical annulus when the other parameters M=0.5, G=1 B_r =1 and E_c =0.5 were kept constant. Results show that increase in the third grade parameter increases the temperature of the system, as the profiles converges to zero when $R = 2$, which is the wall of the outer cylinder. This confirmed   the boundary condition and the isothermal nature of the walls temperature of the outer cylinder.

6.0 Conclusion

In this paper, we applied perturbation technique to analyse the flow of an incompressible MHD third grade fluid through inclined rotating cylindrical pipes with isothermal wall and Joule heating. Results show that the Eckert number, the Grashhof number reduce the velocity of the fluid flow, while the magnetic field parameter, the Grashhof number, the Eckert number and the third grade parameter increase the temperature of the system. Results further show that the temperature profiles converges to zero that is when $R = 2$, confirming the boundary condition, which means that the wall temperature is isothermally stable.

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