

THE MATHEMATICAL MODELING OF HEART BEATS NUMERICALLY

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Abstract:

Each normal heart beat begins with the discharge depolarization of the sinoatrial (SA) node, high up in the right atrium. This is a spontaneous event, occurring 60–100 times every minute. Depolarization of the SA node does not cause any noticeable wave on the standard ECG (although it can be seen on specialized intracardiac recordings). The first detectable wave appears when the impulse spreads from the SA node to depolarize the atria. This produces the P wave. The atria contain relatively little muscle, so the voltage generated by atrial depolarization is relatively small. From the viewpoint of most leads, the electricity appears. The aim of this study depends on collecting, describing and analyzing information using several examples and types of curves theory. Several stages of the proposed models are designed and implemented on the solution programs used in Matlab programming .The quality algorithm was programming . The quality algorithm was used and developed to classification heart beats for accurate results. The proposed model contributed to providing an integrated computer solution for all stage of the classification starting from the stage of classification of heart beats in curve theory and the stage of displaying and representing the results graphically in the Matlab program . this was done by sampled taker ECG from patients to reach accurate readings that provide an ideal model applicable in many of the techniques that contribute to solving our daily problems.

Key wards: *Mathematical , Modeling , Heart Beats, Numerically*

1. INTRODUCTION:

Heart is a hollow muscular organ that pumps blood around the body with each beat; it pumps, at rest, about 70 milliliters of blood considerably more during exercise. Over a 70 year life span and at a rate of around 70 beats per minute, the heart will beat over 2.5 billion times. The heart consists of four main chambers (left and right atria, and left and right ventricles) and four valves (aortic, mitral, pulmonary and tricuspid) venous blood returns to the right atrium via the superior and inferior venae cavae, and leaves the right ventricle for the lungs via the pulmonary artery oxygenated blood from the lungs returns to the left atrium via the four pulmonary veins, and leaves the left ventricle via the aorta.[9]pp1 The electrical discharge for each cardiac cycle normally starts in a special area of the right atrium called the ‘sinoatrial (SA) node’. Depolarization then spreads through the atrial muscle fibres. There is a delay while depolarization spreads through another special area in the atrium, the ‘atrioventricular node’ (also called the ‘AV node’, or sometimes just ‘the node’). Thereafter, the depolarization wave travels very rapidly down specialized conduction tissue, the ‘bundle of His’, which divides in the septum between the ventricles into right and left bundle branches. The left bundle branch itself divides into two. Within the mass of ventricular muscle, conduction spreads somewhat more slowly, through specialized tissue called ‘Purkinje fibres’.[9]pp40

MATLAB is a computer language designed for technical computing, mathematical analysis, and system simulation. It is interactive in nature and is specifically designed to solve problems in the engineering fields, sciences, and business applications and appears to be evolving as the preferred tool in the processes of engineering analysis and synthesis. The MATLAB software was originally developed at the University of New Mexico and Stanford University in the late 1970s. By 1984, a company was established named as Math Works by Jack Little and Cleve Moler with the clear objective of commercializing Math Works over a million engineers and scientists use Math Works to day in well over 3000 universities worldwide and it is considered a standard tool in education, business, and industry.[5]pp5

The matrix and unlike other computer languages it does not have to be dimensioned or declared. MATLAB's original objective was to be the tool to solve mathematical problems in linear algebra, numerical analysis, and optimization but it quickly evolved as the preferred tool for data analysis, statistics, signal processing, control systems, economics, weather forecast, and many other applications. Over the years, Matlab evolved creating an extended library of specialized built-in functions that are used to generate among other things two-dimensional (2-D) and 3-D graphics and animation and offers numerous supplemental packages called tool boxes that provide additional software power in special areas of interest such as curve fitting. MATLAB is a high-level computer language for scientific computing and data visualization built around an interactive programming environment. It is becoming the premiere plot form for scientific computing at educational institutions and debugged quickly, allowing the user to concentrate more on the principles behind the program and less on programming itself since there is no need to compile, link and execute after each correction, Math Works programs can be developed in much shorter time than equivalent FORTRAN or C programs on the negative side MATLAB does not produce stand-alone applications the programs can be run only on computers that have MATLAB installed.[3]pp1

2. Mathematical Modeling:

Definition (2.1):

The matrix laboratory package referred to as Math Works was originally designed to serve as the interactive link to the numerical computation libraries LINPACK and EISPACK that were used by engineers and scientists when they were dealing with sets of equations.[3]pp12

i. The Most Basic Data Structure in Matlab is the Matrix:

- a. A two-dimensional
- b. Rectangular shaped data structure capable of storing multiple elements in an easily accessible format.
- c. These data elements can be numbers, characters, logical states of true or false, or even other Matlab. Structure type Matlab uses these two-dimensional matrices to store single numbers and linear series of numbers as well.[3]pp197

Matlab has a high-level graphics capability that allows users to display data in various forms without having to incorporate extensive information into a command or into scripts. This easy procedure uses default values for graphical objects in Matlab's object-oriented graphics system, Handle Graphics. For more customized and advanced use, the values can be specified or changed on the command line or in the text of m-file scripts; and there is also a point-and-click GUI for users to change object values in order to alter display characteristics.

Display possibilities include 2-D plots, 3-D plots, visual aids such as pie charts and histograms, contours, and animation. In addition, there are many attributes of objects that can be customized including scaling, colors, fonts, perspective angles, lighting, and shading and so forth.[8]pp66

ii. Three Dimensional Plotting :

Matlab has several commands used in displaying data in three dimensions.

Among these are routines for displaying lines with a 3-D perspective, making surface plots of a function of two independent variables and making contour plots.

`>> plot3(x, y, z) % line in three-dimensional space`

`>> mesh(q) % wire frame surface for q(x, y)`

```
>>surf (q) %quadrilateral surface rendering
>>contour%2D projection of equal ln eight lines
```

The plot3 command requires 3 vector arguments, the first two forming a grid of independent variable values and the third being the functional values obtained from the independent variable values.

Thus, all three vectors must be the same length.

In contrast, the argument for the surface plots require that the argument be a matrix whose values correspond to pairs of independent variable values in a grid. Matlab does not come with an experimental data set equivalent to the sun spot data that can be used for illustrating 3 . D plotting.

As an illustration of plotting a line in 3- D space, set each of the three columns of the pop center, clot file to separate vectors and then use those as the arguments for plot3

Command

```
>>clear;
>>load pop center clot ,
>> decade = pop center (: , 1) ;
>>north = pop center (: , 2); [5]pp70
```

3. Heart Beats:

The heart is a hollow muscular organ that pumps blood around the body with each beat ,it pumps ,at rest ,about 70 milliliters of blood and considerably more during exercise over a70 year life span and at a rate of around 70 beats per minute ,the heart will beat over 2.5 billion times. The heart consists of four main chambers (left and right atria, and left and right ventricles) and four valves (aortic, mitral, pulmonary and tricuspid) venous blood returns to the right atrium via the superior and inferior venae cavae, and leaves the right ventricle for the lungs via the pulmonary artery oxygenated blood from the lungs returns to the left atrium via the four pulmonary veins, and leaves the left ventricle via the aorta.

The heart is made up of highly specialized cardiac muscle comprising myocardial cells (myocytes) ,which differs markedly from skeletal muscle because heart muscle

- is under the control of the autonomic nervous system
- contracts in a rhythmic manner
- has a large number of mitochondria which make the myocytes resistant to fatigue

cannot function adequately in anaerobic (ischemic) conditions.[9]pp1

Each normal heartbeat begins with the discharge ('depolarization') of the SA node. The impulse then spreads from the SA node to depolarize the atria. After flowing through the atria, the electrical impulse reaches the AV node, low in the right atrium. Once the impulse has traversed the AV node, it enters the bundle of His which then divides into left and right bundle branches as it passes into the interventricular septum. The right bundle branch conducts the wave of depolarization to the right ventricle, whereas the left bundle branch divides into anterior and posterior fascicles that conduct the wave to the left ventricle. The conducting pathways end by dividing into Purkinje fibres that distribute the wave of depolarization rapidly throughout both ventricles. Normal depolarization of the ventricles is therefore usually very fast, occurring in less than 0.12 ms.[9] pp15

i. The Rhythm of the Heart:

As we shall see later, electrical activation of the heart can sometimes begin in places other than the SA node. The word 'rhythm' is used to refer to the part of the heart which is controlling the activation sequence. The normal heart rhythm, with electrical activation beginning in the SA node, is called 'sinus rhythm.[11]pp23

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ii. Heart Rate :

Measurement of the heart rate and the identification of the cardiac rhythm go hand in hand, as many abnormalities of heart rate result from arrhythmias.

To begin with, however, we will simply describe ways to measure the heart rate and the abnormalities that can affect it. When we talk of measuring the heart rate, we usually mean the *ventricular* rate, which corresponds to the patient's pulse. Depolarization of the ventricles produces the QRS complex on the ECG, and so it is the rate of QRS complexes that needs to be measured to determine the heart rate. Measurement of the heart rate is simple and can be done in several ways. However, before you try; to measure anything, check that the ECG has been recorded at the standard UK and US paper speed of 25 mm/s. If so, then all you have to remember is that a 1-min ECG tracing covers 300 large squares.[9]pp54

If the patient's rhythm is regular, all you have to do is count the number of large squares between two consecutive QRS complexes, and divide it into 300 an alternative, and slightly more accurate, method is to count small squares rather than big ones. For this method, you need to remember that a 1-min ECG tracing covers 1500 small squares. Count the number of small squares between two consecutive QRS complexes, and divide it into 1500. This method does not work so well

when the rhythm is irregular, as the number of large squares between each QRS complex varies from beat to beat. So, instead, count the number of QRS complexes in 50 large squares (Fig. 3.2) – the length of the rhythm strip on a standard ECG. This is the number of QRS complexes in 10 s.[6]pp47

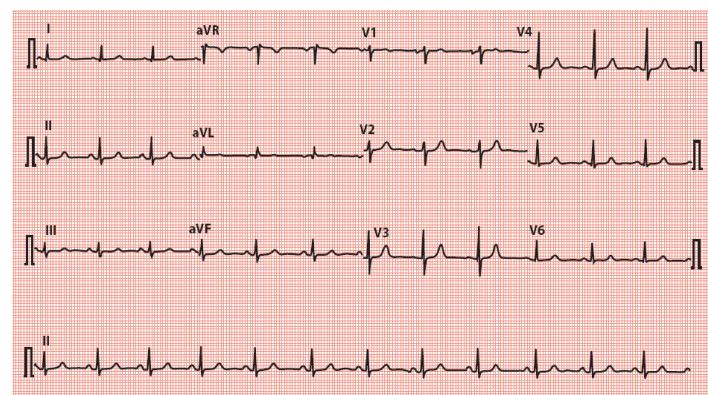


Figure No '3, 1' Calculating Heart Rate when the Rhythm is Regular

Key Point:

-There are approximately 4 large squares between each QRS complex, corresponding to a heart rate of approximately 75/min. - more precisely, there are 21 small squares between each QRS complex, giving a more accurate heart rate of 71/min

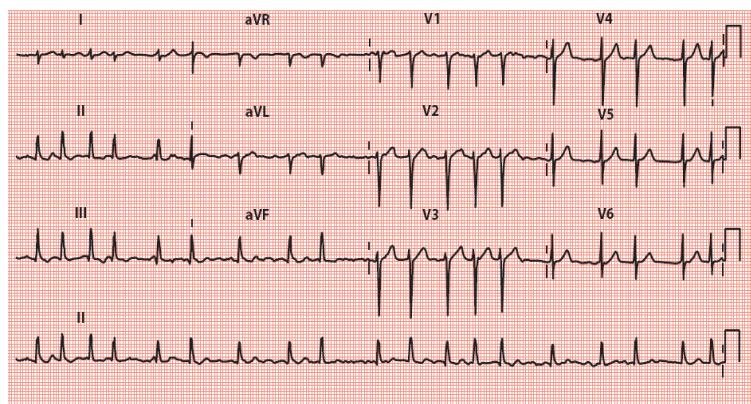


Figure No '3, 2' Calculating Heart Rate when the Rhythm is Irregular

Key Point :

There are 19 QRS complexes in 50 large squares (10s) corresponding to a heart rate of 114/min. An electrocardiogram ruler can be helpful, but follow the instructions on it carefully. [6] pp 49

4. Discussion and Results:

We gave computer solution by using Matlab presented a convertor of ECG patients by using powerful algorithm.

i. Classifications of Heart Beats by Matlab Solutions :

Used Normalized Cross-Correlation (NCC) techniques in matlab program for classifications of hearts beats to normal or upnormal.

Solution:

```
[a b]=uigetfile({'*.*'});
RGB=imread([b a]);
I = rgb2gray(RGB);
% I=(I);
imshow(I)
[a b]=uigetfile({'*.*'});
RGB=imread([b a]);
J = rgb2gray(RGB);
% I=(I);
imshow(J)
imshowpair(I,J,'montage')
c = normxcorr2(J(:,:,1),I(:,:,1));
figure, surf(c), shading flat
[max_c, imax] = max (abs(c(:)));
```

```

[ypeak, xpeak] = ind2sub (size(c),imax(1));
corr_offset = [(xpeak-size (J, 2))
(ypeak-size (J, 1))];

offset = corr_offset;
xoffset = offset(1);
yoffset = offset(2);
xbegin = round(xoffset+1);
xend = round(xoffset+ size(J,2));
ybegin = round(yoffset+1);
yend = round(yoffset+size(J,1));
[ypeak, xpeak] = find(c==max(c(:)));
yoffSet = ypeak-size(J,1);
xoffSet = xpeak-size(J,2);
hFig = figure;
hAx = axes;
imshow(I,'Parent', hAx);
imrect(hAx, [xoffSet+1, yoffSet+1, size(J,2), size(J,1)]);
if (max_c>= 0.7)
('State of ECG Normal')
else
('State of ECG Upnormal')
end
Result:
ans =
State of ECG Normal
Represent the Solution Graphically:

```



Figure No '4.1': Template Patient Normal ECG

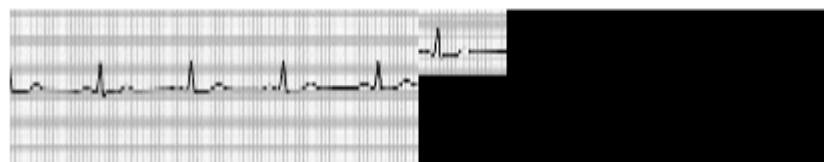


Figure No '4.2': Scan Template Patient Normal ECG on Patients ECG

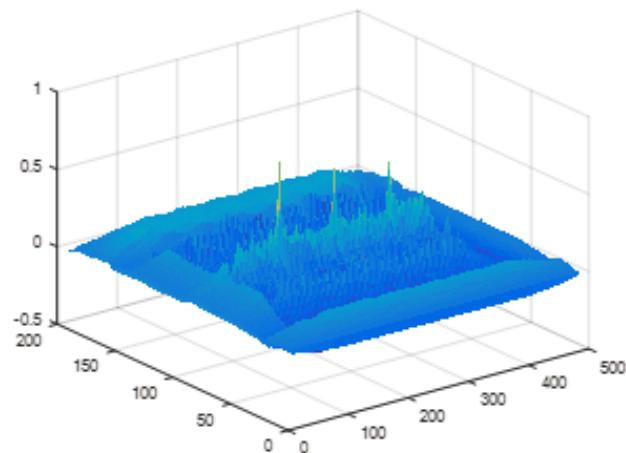


Figure No '4.3': The Peak of the NCC

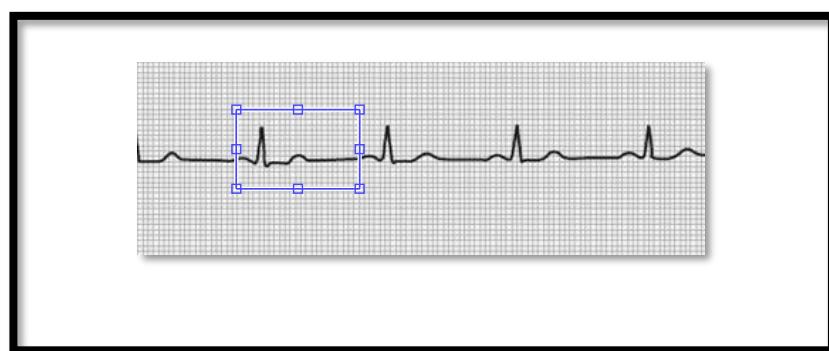


Figure No .”4.4”: Classification of Heart Beats patient by Using NCC.

Results:

After implementing all the steps described in the previous, the model is evaluated, and the final results are discussed. Results obtained from the proposed model automatically during implementation, and testing process to get accurate results by used MATLAB by applying this is to repeat the experiment several times using the feet and non-feet patients ECGs by using Normalized Cross-Correlation (NCC) as each investigation resulted in a curve of graphics. The mean was calculated for accuracy to obtain excellent accuracy due to repetition, training, and verification in each time.

Discussion:

The performance of the computer solution model for patient's ECGs was evaluated in this study through the graphics presented in the previous to verify its validity. This study used a new method to solve of patients ECGs by make powerful algorithm for results and represent automatically in graphics unlike other studies that focused on a calculation by differential equations and others, such as the linear system of smooth curves etc.

This study achieved great success in solving different sex old for patient's ECGs. But it failed to direct these solutions into the application in the problems of patient itself, as it was satisfied only with the solution and represented it graphically. This problem was solved by increasing the solution of more different Template Patient Normal ECG for children, women and men and thus the model achieved an accuracy of more than 98.5% and the presence classifications of heart beats in curve theory similarities were found between this study and others in the computerized solution using MATLAB.. This study was characterized by presenting a complete solution by designing an algorithm that performs the solution and extracts the results and represents them graphically. That is, this study was applied to the computerized solution procedure on patient's ECGs and proofs in the solution stage, the results stage, and the results presentation stage .

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