Low Separation Axioms Via (1, 2)*-$M_{m\pi}$-Closed Sets In Biminimal Spaces

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Abstract: The purpose of this paper is to introduce the concepts of (1, 2)*-$M_{m\pi}$- T_0 space, (1, 2)*- $M_{m\pi}$ - T_1 space and (1, 2)*- $M_{m\pi}$ - T_2 space in a biminimal spaces. We study some of the characterizations and properties of these separation axioms. Further we discuss (1, 2)*-$M_{m\pi}$ -R_0 and (1, 2)*- $M_{m\pi}$ -R_1 spaces in biminimal spaces. The implications of these axioms among themselves are also investigated

Key Words & Phrases: (1, 2)*-$M_{m\pi}$- T_0, (1, 2)*- $M_{m\pi}$ - T_1, (1, 2)*- $M_{m\pi}$ - T_2, (1, 2)*- $M_{m\pi}$ - R_0, (1, 2)*- $M_{m\pi}$ -R_1.

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1. INTRODUCTION

The concept of minimal structure (briefly m-structure) was introduced by V. Popa and T. Noiri [12] in 2000. Also they introduced the notion of m_X-open set and m_X-closed set and characterize those sets using m_X-closure and m_X- interior operators respectively. Further they introduced M-continuous functions and studied some of its basic properties. The separation axioms R_0 and R_1 were introduced and studied by N. A. Shanin [15] and C. T. Yang [16]. In 1963, they were rediscovered by A. S. Davis [4]. In literature, [1, 2, 3, 4, 6, 7, 9, 10, 11, 12] many authors introduced various separation axioms. Recently, Ravi et al [13, 14] studied $\tau_{1,2}$-open sets in biminimal spaces. In this paper we introduce and study some separation axioms in a biminimal structure space.

2. PRELIMINARIES

We recall the following definitions which are useful in the sequel.

Definition: 2. 1. [5] Let X be a non-empty set and þ(X) the power set of X. A sub family m_x of þ(X) is called a minimal structure (briefly m-structure) on X if $\varphi \in m_x$ and $X \in m_x$.

Definition: 2. 2. [13] A set X together with two minimal structures m_x^1 and m_x^2 is called a biminimal space and is denoted by (X, m_x^1, m_x^2).

Throughout this paper, (X, m_x^1, m_x^2) (or X) denote biminimal structure space.

Definition: 2. 3. [13] Let S be a subset of X. Then S is said to be $m_x^{(1, 2)*}$-open if $S=A \cup B$ where $A \in m_x^1$ and $B \in m_x^2$. The complement of $m_x^{(1, 2)*}$-open set is called $m_x^{(1, 2)*}$-closed set. The family of all $m_x^{(1, 2)*}$-open (resp. $m_x^{(1, 2)*}$-closed) subsets of X is denoted by $m_x^{(1, 2)*}$-O(X) (resp. $m_x^{(1, 2)*}$-C(X)).

Definition: 2. 4. [13] Let S be a subset of X. Then

1. the $m_x^{(1, 2)*}$-interior of S denoted by $m_x^{(1, 2)*}$-int(S) is defined by $\cup\{G: G \subseteq S$ and G is $m_x^{(1, 2)*}$-open$\}$.
2. the $m_x^{(1, 2)*}$-closure of S denoted by $m_x^{(1, 2)*}$-cl(S) is defined by $\cap\{F: S \subseteq F$ and F is $m_x^{(1, 2)*}$-closed$\}$.

Definition: 2. 5. [14] A subset A of X is called regular -$m_x^{(1, 2)*}$-open if A= $m_x^{(1, 2)*}$-int ($m_x^{(1, 2)*}$-cl (A)).

Definition 2. 6. [8] The finite union of regular -$m_x^{(1, 2)*}$-open set in X is called $m_x^{(1, 2)*}$-π-open set.

Definition 2. 7. [8] A subset A of X is said to be $m^{(1, 2)*}$-πg-closed set if $m_x^{(1,2)*}$-cl (A) $\subseteq$ G whenever $A \subseteq G$ and G is $m_x^{(1,2)*}$-π-open set. The complement of an $m^{(1, 2)*}$-πg-closed set is called $m^{(1, 2)*}$-πg-open set.

The family of all $m^{(1, 2)*}$-πg-open (resp. $m^{(1, 2)*}$-πg-closed) subsets of X is denoted by $m^{(1, 2)*}$-πg-O(X) (resp. $m^{(1, 2)*}$-πg-C(X)).

Definition 2. 8. [8] A subset A of X is said to be (1, 2)*- $M_{m\pi}$-closed set if $m_x^{(1,2)*}$-cl (A) $\subseteq$ G whenever $A \subseteq G$ and G is $m^{(1, 2)*}$-πg-open set. The complement of an (1, 2)*- $M_{m\pi}$-closed set is called a (1, 2)*- $M_{m\pi}$-open set in X.

The family of all (1, 2)*- $M_{m\pi}$-open (resp. (1, 2)*- $M_{m\pi}$-closed) subsets of X is denoted by (1, 2)*- $M_{m\pi}$-O(X) (resp. (1, 2)*- $M_{m\pi}$-C(X)).

3. (1, 2)*- $M_{m\pi}$- SEPARATION AXIOMS:

Definition 3. 1. The union of all (1, 2)*- $M_{m\pi}$-open sets in a biminimal space X, which are contained in a subset A of X is called the (1, 2)*- $M_{m\pi}$-interior of A and is denoted by (1, 2)*- $M_{m\pi}$-int (A).

Definition 3. 2. The (1, 2)*- $M_{m\pi}$-closure of A of X is the intersection of all (1, 2)*- $M_{m\pi}$-closed sets that contains A and is denoted by (1, 2)*- $M_{m\pi}$-cl (A).

Definition 3. 3. A biminimal space X is called (1, 2)*- $M_{m\pi}$ -T_0 (resp. $m^{(1, 2)^*}$- πg -T_0) space if for any two distinct points x, y in X, there exists a (1, 2)*- $M_{m\pi}$ – open ($m^{(1, 2)^*}$- πg –open) set containing only one of x and y but not the other.

Clearly, every (1, 2)*- $M_{m\pi}$ -T_0 space is a $m^{(1, 2)^*}$- πg -T_0 space, since every (1, 2)*- $M_{m\pi}$ – open set is a $m^{(1, 2)^*}$- πg –open set. The converse is not true in general.

Example 3. 4. Let X = {a, b, c}, τ_1 = {φ, X, {b}, {a, b}} and τ_2 = {φ, X, {b, c}}. Then $m^{(1, 2)^*}$-πgO(X) = {φ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}} and (1, 2)*- $M_{m\pi}$-O(X) = {φ, X, {b}, {a, b}, {b, c}}. Therefore, X is $m^{(1, 2)^*}$- πg -T_0, but not (1, 2)*- $M_{m\pi}$ -T_0 space**.**

Theorem 3. 5. If (1, 2)*- $M_{m\pi}$ -closures of distinct points are distinct in any biminimal space X, then it is (1, 2)*- $M_{m\pi}$ -T_0.

Proof. Let x, y ∈ X, x ≠ y. By the hypothesis, (1, 2)*- $M_{m\pi}$ –cl ({x}) ≠ (1, 2)*- $M_{m\pi}$ –cl ({y}). Then, there exists a point z Є X such that z belongs to exactly one of the two sets, say (1, 2)*- $M_{m\pi}$ –cl ({y}) but not to (1, 2)*- $M_{m\pi}$ –cl ({x}). If y Є (1, 2)*- $M_{m\pi}$ –cl ({x}), then (1, 2)*- $M_{m\pi}$ – cl ({y}) ⊆ (1, 2)*- $M_{m\pi}$ –cl ({x}) which implies z Є (1, 2)*- $M_{m\pi}$ –cl ({x}), a contradiction. So y ∈ X - (1, 2)*- $M_{m\pi}$ –cl ({x}), a (1, 2)*- $M_{m\pi}$ –open set which does not contain x. This shows that X is (1, 2)*- $M_{m\pi}$ –T_0.

Theorem 3. 6. In any biminimal space X, (1, 2)*- $M_{m\pi}$ -closures of distinct points are distinct.

Proof. Let x, y ∈ X, x ≠ y. **Case (a):** {x} is $m_x^{(1, 2)^*}$ -closed. Then {x} is (1, 2)*- $M_{m\pi}$ -closed. Now y ≠ x implies y ∉ {x} = (1, 2)*- $M_{m\pi}$ -cl ({x}). Hence (1, 2)*- $M_{m\pi}$ –cl ({y}) ≠ (1, 2)*- $M_{m\pi}$ – cl ({x}). **Case (b):** {x} is not $m_x^{(1, 2)^*}$ -closed. Then X- {x} is not $m_x^{(1, 2)^*}$ - open and therefore, X is only $m_x^{(1, 2)^*}$ -open set containing X – {x}. Hence X – {x} is (1, 2)*- $M_{m\pi}$ –closed set. Now y ∈ X- {x} implies (1, 2)*- $M_{m\pi}$ -cl ({y}) ⊆ X –{x}. Hence x ∉ (1, 2)*- $M_{m\pi}$ –cl ({y}) and (1, 2)*- $M_{m\pi}$ –cl ({y}) ≠ (1, 2)*- $M_{m\pi}$ –cl ({x}).

Theorem 3. 7. Every biminimal space is (1, 2)*- $M_{m\pi}$ -T_0.

Proof. Follows from Theorem 3. 5. and Theorem 3. 6.

Definition 3. 8. A biminimal space X is called a (1, 2)*- $M_{m\pi}$ -C_0 space if for any two distinct points x, y in X, there exists a (1, 2)*- $M_{m\pi}$ -open set such that (1, 2)*- $M_{m\pi}$ –cl (G) contains one of x and y, but not the other.

Theorem 3. 9. If a biminimal space X is (1, 2)*- $M_{m\pi}$ - C_0 then it is (1, 2)*- $M_{m\pi}$ –T_0.

Proof. Let X be (1, 2)*- $M_{m\pi}$ - C_0 and x, y Є X with $x \neq y$. Then there exists a (1, 2)*- $M_{m\pi}$ –open set G of X such that $x \in$ (1, 2)*- $M_{m\pi}$ –cl (G) and $y \notin$ (1, 2)*- $M_{m\pi}$ –cl(G). Since G is (1, 2)*- $M_{m\pi}$ – open, (1, 2)*- $M_{m\pi}$ –cl (G) is also (1, 2)*- $M_{m\pi}$ –open. Moreover $x \in$ (1, 2)*- $M_{m\pi}$ –cl (G) and $y \notin$ (1, 2)*- $M_{m\pi}$ –cl (G). Hence X is (1, 2)*- $M_{m\pi}$ -T_0.

Definition 3. 10. A biminimal space X is said to be (1, 2)*- $M_{m\pi}$ -T_1 if for any two distinct points x, y in X, there exists a pair of (1, 2)*- $M_{m\pi}$ –open sets, one containing x but not y and the other containing y but not x.

Definition 3. 11. A biminimal space X is said to be (1, 2)*- $M_{m\pi}$ -C_1 if for any two distinct points x, y in X, there exists U, V Є (1, 2)*- $M_{m\pi}$ –O(X), such that (1, 2)*- $M_{m\pi}$ –cl (U) containing x but not y and (1, 2)*- $M_{m\pi}$ –cl (V) containing y but not x.

Remark 3. 12.

1. Every (1, 2)*- $M_{m\pi}$ -T_1 space is (1, 2)*- $M_{m\pi}$ -T_0.
2. Every (1, 2)*- $M_{m\pi}$ -C_1 space is (1, 2)*- $M_{m\pi}$ –T_1.
3. Every (1, 2)*- $M_{m\pi}$ -C_1 space is (1, 2)*- $M_{m\pi}$ –C_0.

But the converses are not true in general as illustrated in the next example.

Example 3. 13.

1. Let X = {a, b, c}, τ_1 = {φ, X, {b}} and τ_2 = {φ, X, {c}}. Then (1, 2)*- $M_{m\pi}$-O(X) = {φ, X, {b}, {c}, {b, c}}. It is clear that, X is (1, 2)*- $M_{m\pi}$ -T_0, but not (1, 2)*- $M_{m\pi}$ –T_1 space.
2. Let X = {a, b, c}, τ_1 = {φ, X, {a}} and τ_2 = {φ, X, {b}}. Then (1, 2)*- $M_{m\pi}$-O(X) = {φ, X, {a}, {b}, {a, b}}. Here X is (1, 2)*- $M_{m\pi}$ -T_0, but not (1, 2)*- $M_{m\pi}$ –C_0 space.
3. Let X = {a, b, c, d}, τ_1 = {φ, X, {a}, {b}} and τ_2 = {φ, X, {a, b, d}}. Then (1, 2)*- $M_{m\pi}$-O(X) = {φ, X, {a}, {b}, {a, b}, {a, b, d}}. Then, X is (1, 2)*- $M_{m\pi}$ –C_0, but not (1, 2)*- $M_{m\pi}$ –C_1 space.

Theorem 3. 14. In a biminimal space X, the following statements are equivalent.

1. X is (1, 2)*- $M_{m\pi}$ -T_1.
2. Each one point set is (1, 2)*- $M_{m\pi}$ -closed set in X.

Proof. (1) => (2). Let X be (1, 2)*- $M_{m\pi}$ -T_1 and $x \in X$. Suppose (1, 2)*- $M_{m\pi}$ –cl ({x}) ≠ {x}. Then we can find an element $y \in$ (1, 2)*- $M_{m\pi}$ –cl({x}) with $y \neq x$. Since X is (1, 2)*- $M_{m\pi}$ –T_1, there exist (1, 2)*- $M_{m\pi}$ –open sets U and V such that $x \in U$, $y \notin U$ and y Є V, $x \notin V$. Now $x \in$

V^C and V^C is (1, 2)*- $M_{m\pi}$ –closed set. Therefore, (1, 2)*- $M_{m\pi}$ –cl ({x}) $\subseteq$ V^C which implies y $\in V^C$, a contraction. Hence, (1, 2)*- $M_{m\pi}$ –cl ({x}) = {x} or {x} is (1, 2)*- $M_{m\pi}$ –closed.

(2) => (1). Let x, y$\in$ X and x $\neq$ y. Then {x} and {y} are (1, 2)*- $M_{m\pi}$ –closed. Therefore, U = $(\{y\})^C$ and V = $(\{x\})^C$ are (1, 2)*- $M_{m\pi}$ – open and x $\in$ U, y $\notin$ U and y $\in$ V, x $\notin$ V. Hence is (1, 2)*- $M_{m\pi}$ -T_1.

Definition 3. 15. A biminimal space X is called a (1, 2)*- $M_{m\pi}$ -T_2 space if for any two distinct points x, y in X, there exists a pair of disjoint (1, 2)*- $M_{m\pi}$ –open sets U and V such that x $\in$ U and y $\in$ V and U ∩ V = φ.

Definition: 3. 16 A function f: X→Y is called (1, 2)*-$M_{m\pi}$-irresolute if the inverse image of every (1, 2)*-$M_{m\pi}$-closed set in Y is (1, 2)*-$M_{m\pi}$-closed set in X.

Theorem 3. 17. If f: X → Y is an injective, (1, 2)*- $M_{m\pi}$ -irresolute function and Y is (1, 2)*- $M_{m\pi}$ -T_2 then X is (1, 2)*- $M_{m\pi}$ -T_2.

Proof. Let x, y $\in$ X and x $\neq$ y. Since f is injective, f(x) $\neq$ f(y) in Y and there exist disjoint (1, 2)*- $M_{m\pi}$ -open sets U, V such that f(x) $\in$ U and f(y) $\in$ V. Let G=f^{-1}(U) and H=f^{-1}(V). Then x$\in$ G, y $\in$ H and G, H $\in$ (1, 2)*- $M_{m\pi}$ -O(X). Also G ∩ H = f^{-1}(U) ∩ f^{-1}(V) = f^{-1}(U ∩V) = φ. Thus X is (1, 2)*- $M_{m\pi}$ -T_2.

Theorem 3. 18. If f: X → Y is an injective, (1, 2)*- $M_{m\pi}$ -irresolute function and Y is (1, 2)*- $M_{m\pi}$ –T_1 then X is (1, 2)*- $M_{m\pi}$ –T_1.

Proof. The proof is similar to the above theorem.

Remark 3. 19. Every (1, 2)*- $M_{m\pi}$ –T_2 space is (1, 2)*- $M_{m\pi}$ –T_1.

Definition 3. 20. A biminimal space X is called a (1, 2)*- $M_{m\pi}$ –R_0 space if for each (1, 2)*- $M_{m\pi}$ –open set G, x $\in$ G, implies (1, 2)*- $M_{m\pi}$ –cl ({x}) $\subseteq$ G.

Theorem 3. 21 : For any biminimal space X, the following are equivalent:

1. X is (1, 2)*- $M_{m\pi}$-R_0.
2. F $\in$ (1, 2)*- $M_{m\pi}$-C(X) and x $\notin$ F => F $\subseteq$ U and x $\notin$ U for some U $\in$ (1, 2)*- $M_{m\pi}$-O(X).
3. F $\in$ (1, 2)*- $M_{m\pi}$-C(X) and x $\notin$ F => F ∩ (1, 2)*- $M_{m\pi}$-cl ({x}) = φ.
4. For any two distinct points x, y of X, either (1, 2)*- $M_{m\pi}$-cl ({x}) = (1, 2)*- $M_{m\pi}$-cl ({y}) or (1, 2)*- $M_{m\pi}$-cl ({x}) ∩ (1, 2)*- $M_{m\pi}$-cl ({y}) = φ.

Proof. 1 => 2: F $\in$ (1, 2)*- $M_{m\pi}$-C(X) and x $\notin$ F => x $\in$ X\ F $\in$ (1, 2)*- $M_{m\pi}$-O(X) => (1, 2)*- $M_{m\pi}$-cl ({x}) $\subseteq$ X \ F (by (1)). Put U = X \ (1, 2)*- $M_{m\pi}$-cl ({x}). Then x $\notin$ U $\in$ (1, 2)*- $M_{m\pi}$-O(X) and F $\subseteq$ U.

2=> 3: F $\in$(1, 2)*- $M_{m\pi}$-C(X) and x $\notin$F => there exists U $\in$(1, 2)*- $M_{m\pi}$-O(X) such that x $\notin$U and F $\subseteq$U (by (2)) => U ∩ (1, 2)*- $M_{m\pi}$-cl ({x}) = φ => F ∩ (1, 2)*- $M_{m\pi}$-cl ({x}) = φ.

3=> 4: Suppose that for any two distinct points x, y of X, (1, 2)*- $M_{m\pi}$-cl ({x}) ≠ (1, 2)*- $M_{m\pi}$-cl ({y}). Then suppose without any loss of generality that there exists some z $\in$(1, 2)*- $M_{m\pi}$-cl ({x}) such that z $\notin$(1, 2)*- $M_{m\pi}$-cl ({y}). Thus there exists V$\in$(1, 2)*- $M_{m\pi}$-O(X) such that z $\in$V and y$\notin$V but x$\in$V. Thus x$\notin$(1, 2)*- $M_{m\pi}$-cl ({y}). Hence by (3), (1, 2)*- $M_{m\pi}$-cl ({x}) ∩ (1, 2)*- $M_{m\pi}$-cl ({y}) = φ.

4=> 1 : Let U$\in$(1, 2)*- $M_{m\pi}$-O(X) and x $\in$U. Then for each y $\notin$U, x $\notin$(1, 2)*- $M_{m\pi}$-cl ({y}). Thus (1, 2)*- $M_{m\pi}$-cl ({x}) ≠ (1, 2)*- $M_{m\pi}$-cl ({y}). Hence by (4), (1, 2)*- $M_{m\pi}$-cl ({x}) ∩ (1, 2)*- $M_{m\pi}$-cl ({y}) = φ, for each y $\in$X \ U. So (1, 2)*- $M_{m\pi}$-cl ({x}) ∩ [∪ {(1, 2)*- $M_{m\pi}$-cl ({y}) : y $\in$X \ U}] = φ ………….(i).

Now, U$\in$(1, 2)*- $M_{m\pi}$-O(X) and y $\in$X \ U => {y} $\subseteq$(1, 2)*- $M_{m\pi}$-cl ({y}) $\subseteq$(1, 2)*- $M_{m\pi}$-cl (X \ U) = X \ U.Thus X \ U = ∪ {(1, 2)*- $M_{m\pi}$-cl ({y}) : y $\in$X \ U}. Hence from (i), (1, 2)*- $M_{m\pi}$-cl ({x}) ∩(X \ U) = φ => (1, 2)*- $M_{m\pi}$-cl ({x}) $\subseteq$U, showing that X is (1, 2)*- $M_{m\pi}$-R_0.

Definition 3. 22. A biminimal space X is called a (1, 2)*- $M_{m\pi}$ –R_1 space if for any two distinct points x, y in X, with (1, 2)*- $M_{m\pi}$ -cl ({x}) ≠ (1, 2)*- $M_{m\pi}$ –cl ({y}), there exists pair of disjoint (1, 2)*- $M_{m\pi}$ –open sets U and V such that (1, 2)*- $M_{m\pi}$ –cl ({x}) $\subseteq$U and (1, 2)*- $M_{m\pi}$ –cl ({y}) $\subseteq$V.

Theorem 3. 23. Every (1, 2)*- $M_{m\pi}$ –R_1 biminimal space is (1, 2)*- $M_{m\pi}$ –R_0.

Proof. Let X be (1, 2)*- $M_{m\pi}$ –R_1 and let G be a (1, 2)*- $M_{m\pi}$ –open set containing x. If (1, 2)*- $M_{m\pi}$ –cl ({x}) $\nsubseteq$ G then there exists an element y ∈ (1, 2)*- $M_{m\pi}$ –cl ({x}) ∩ G^C. Since G^C is (1, 2)*- $M_{m\pi}$ –closed, (1, 2)*- $M_{m\pi}$ –cl ({y}) $\subseteq$ G^C. Now (1, 2)*- $M_{m\pi}$ –cl ({x}) ≠ (1, 2)*- $M_{m\pi}$ –cl ({y}) and X is (1, 2)*- $M_{m\pi}$ –R_1. Hence there exists disjoint (1, 2)*- $M_{m\pi}$ –open sets containing (1, 2)*- $M_{m\pi}$ –cl ({x}) and (1, 2)*- $M_{m\pi}$ –cl ({y}) respectively. This is not possible, since y ∈ (1, 2)*- $M_{m\pi}$ –cl ({x}) ∩(1, 2)*- $M_{m\pi}$ –cl ({y}).

Theorem 3. 24. Let X be a biminimal space. Then X is (1, 2)*- $M_{m\pi}$ -R_0 if and only if for every (1, 2)*- $M_{m\pi}$ -closed set K and x $\notin$K, there exists a (1, 2)*- $M_{m\pi}$ -open set S such that K $\subset$S and x $\notin$S.

Proof. Necessity. Let X be a (1, 2)*- $M_{m\pi}$ -R_0 space and K be a (1, 2)*- $M_{m\pi}$ -closed subset such that x $\notin$K. We have X \ K is (1, 2)*- $M_{m\pi}$ -open and x $\in$X \ K. Since X is (1, 2)*- $M_{m\pi}$

-R0, then (1, 2)*- $M_{m\pi}$ -cl ({x}) $\subset$ X \ K. We obtain K $\subset$ X \ (1, 2)*- $M_{m\pi}$ -cl ({x}). Take S = X \ (1, 2)*- $M_{m\pi}$ -cl ({x}). Thus, S is a (1, 2)*- $M_{m\pi}$ -open set such that K $\subset$ S and x $\notin$ S.

Sufficiency. Let S be a (1, 2)*- $M_{m\pi}$ -open set and x $\in$ U. Then X \ S is a (1, 2)*- $M_{m\pi}$ -closed set and x $\notin$ X \ S. Then there exists a (1, 2)*- $M_{m\pi}$ -open subset U such that X \ S $\subset$ U and x $\notin$ U. We obtain X \ U $\subset$ S and x $\in$ X \ U. Since X \ U is a (1, 2)*- $M_{m\pi}$ -closed set, then (1, 2)*- $M_{m\pi}$ -cl ({x}) $\subset$ X \ U $\subset$ S. Hence, X is a (1, 2)*- $M_{m\pi}$ -R_0 space.

Theorem 3. 25. Let X be a biminimal space. Then X is (1, 2)*- $M_{m\pi}$ –T_1 if and only if it is a (1, 2)*- $M_{m\pi}$ – T_0 and (1, 2)*- $M_{m\pi}$ – R_0.

Proof. Let X be a (1, 2)*- $M_{m\pi}$ – T_1 space. By the definition of (1, 2)*- $M_{m\pi}$ – T_1 space, it is a (1, 2)*- $M_{m\pi}$ – T_0 and (1, 2)*- $M_{m\pi}$ – R_0 space.

Conversely, let X be a (1, 2)*- $M_{m\pi}$ – T_0 space and (1, 2)*- $M_{m\pi}$ – R_0 space. Let x, y be any two distinct points of X. Since X is (1, 2)*- $M_{m\pi}$ – T_0, then there exists a (1, 2)*- $M_{m\pi}$ – open set U such that x $\in$ U and y $\notin$ U or there exists a (1, 2)*- $M_{m\pi}$ –open set V such that y $\in$ V and x $\notin$ V. Let x $\in$ U and y $\notin$ U. Since X is (1, 2)*- $M_{m\pi}$ – R_0, then (1, 2)*- $M_{m\pi}$ –cl ({x}) $\subset$ U. We have y $\notin$ U and then y $\notin$ (1, 2)*- $M_{m\pi}$ –cl ({x}). We obtain y $\in$ X \ (1, 2)*- $M_{m\pi}$ – cl ({x}). Take S = X \ (1, 2)*- $M_{m\pi}$ –cl ({x}). Thus, U and S are (1, 2)*- $M_{m\pi}$ –open sets containing x and y, respectively, such that y $\notin$ U and x $\notin$ S. Hence, X is (1, 2)*- $M_{m\pi}$ – T_1.

Theorem 3. 26. Let X be a biminimal space. Then X is a (1, 2)*- $M_{m\pi}$ -R_0 space if and only if for any x and y in X, (1, 2)*- $M_{m\pi}$ –cl({x}) $\neq$ (1, 2)*- $M_{m\pi}$ –cl({y}) implies (1, 2)*- $M_{m\pi}$ –cl({x}) ∩ (1, 2)*- $M_{m\pi}$ –cl({y}) = φ.

Proof. Let X be (1, 2)*- $M_{m\pi}$ -R_0 and x, y $\in$ X such that (1, 2)*- $M_{m\pi}$ –cl ({x}) $\neq$ (1, 2)*- $M_{m\pi}$ –cl ({y}). Then, there exist a k $\in$ (1, 2)*- $M_{m\pi}$ –cl ({x}) such that k $\notin$ (1, 2)*- $M_{m\pi}$ –cl ({y}) (or k $\in$ (1, 2)*- $M_{m\pi}$ –cl ({y}) such that k $\notin$ (1, 2)*- $M_{m\pi}$ –cl ({x}) and then there exists V $\in$ (1, 2)*- $M_{m\pi}$ –O(X) such that y $\notin$ V and k $\in$ V and hence x $\in$ V. Thus, x $\notin$ (1, 2)*- $M_{m\pi}$ –cl ({y}) and x $\in$ X \ (1, 2)*- $M_{m\pi}$ –cl ({y}) $\in$ (1, 2)*- $M_{m\pi}$ –O(X). We have (1, 2)*- $M_{m\pi}$ –cl ({x}) $\subset$ X \ (1, 2)*- $M_{m\pi}$ –cl ({y}) and (1, 2)*- $M_{m\pi}$ –cl ({x}) ∩ (1, 2)*- $M_{m\pi}$ –cl ({y}) = φ.

Conversely, Let V $\in$ (1, 2)*- $M_{m\pi}$ –O(X) and x $\in$ V. Let y $\notin$ V. We have y $\in$ X \ V. Then x $\neq$ y and x $\notin$ (1, 2)*- $M_{m\pi}$ –cl({y}). We obtain (1, 2)*- $M_{m\pi}$ –cl({x}) $\neq$ (1, 2)*- $M_{m\pi}$ –cl({y}) and then (1, 2)*- $M_{m\pi}$ –cl({x}) ∩ (1, 2)*- $M_{m\pi}$ –cl({y}) = φ. Thus, y $\notin$ (1, 2)*- $M_{m\pi}$ –cl({x}) and then (1, 2)*- $M_{m\pi}$ –cl({x}) $\subset$ V. We obtain that X is a (1, 2)*- $M_{m\pi}$ – R_0 space.

Theorem 3. 27. Let X be a biminimal space. Then the following properties are equivalent:

1. X is a (1, 2)*- $M_{m\pi}$ -R_0 space.

2. x $\in$ (1, 2)*- $M_{m\pi}$ -cl ({y}) if and only if y $\in$ (1, 2)*- $M_{m\pi}$ -cl ({x}) for any points x and y in X.

Proof. 1 =>2. Let X be (1, 2)*- $M_{m\pi}$ -R_0. Let x $\in$ (1, 2)*- $M_{m\pi}$ -cl ({y}) and S be any (1, 2)*- $M_{m\pi}$ –open set such that y $\in$ S. By (1), x $\in$ S. Hence, every (1, 2)*- $M_{m\pi}$ -open set which contains y contains x and then y $\in$ (1, 2)*- $M_{m\pi}$ -cl ({x}).

2 => 1. Let U be a (1, 2)*- $M_{m\pi}$ -open set and x $\in$ U. If y $\notin$ U, then x $\notin$ (1, 2)*- $M_{m\pi}$ -cl ({y}) and hence y $\notin$ (1, 2)*- $M_{m\pi}$ -cl ({x}). We have (1, 2)*- $M_{m\pi}$ -cl ({x}} $\subset$ U. Thus, X is (1, 2)*- $M_{m\pi}$ -R_0.

Theorem 3. 28. The following are equivalent in a biminimal space X.

1. X is (1, 2)*- $M_{m\pi}$ -T_2.
2. X is (1, 2)*- $M_{m\pi}$ –R_1 and (1, 2)*- $M_{m\pi}$ –T_1.
3. X is (1, 2)*- $M_{m\pi}$ –R_1 and (1, 2)*- $M_{m\pi}$ –T_0.

Proof. 1=> 2: X is (1, 2)*- $M_{m\pi}$ –T_2 implies X is (1, 2)*- $M_{m\pi}$ –T_1 and therefore by Theorem 3. 11, every singleton set in X is (1, 2)*- $M_{m\pi}$ –closed. Let x, y $\in$ X and x $\neq$ y. Since X is (1, 2)*- $M_{m\pi}$ -T_2, there exist two disjoint (1, 2)*- $M_{m\pi}$ –open sets U and V containing x and y respectively. Since {x} and {y} are (1, 2)*- $M_{m\pi}$-closed, X is (1, 2)*- $M_{m\pi}$ –R_1.

2 => 3: This is obvious, since X is (1, 2)*- $M_{m\pi}$ –T_1 implies X is (1, 2)*- $M_{m\pi}$ –T_0.

3 => 1: Let x, y $\in$ X and x $\neq$ y.

Case (a). (1, 2)*- $M_{m\pi}$ –cl ({x}) $\neq$ (1, 2)*- $M_{m\pi}$ –cl ({y}). Since X is (1, 2)*- $M_{m\pi}$ –R_1, there exist two disjoint (1, 2)*- $M_{m\pi}$ –open sets U and V such that U $\supseteq$ (1, 2)*- $M_{m\pi}$ –cl ({x}) and V $\supseteq$ (1, 2)*- $M_{m\pi}$ –cl ({y}). Then x $\in$ U and y $\in$ V.

Case (b). (1, 2)*- $M_{m\pi}$ –cl ({x}) = (1, 2)*- $M_{m\pi}$ –cl ({y}). Since x $\neq$ y and X is (1, 2)*- $M_{m\pi}$ –T_0, there exists a (1, 2)*- $M_{m\pi}$ –open sets U containing x but not y. Then y $\in$ U^c, a (1, 2)*- $M_{m\pi}$ –closed set. This implies (1, 2)*- $M_{m\pi}$ –cl ({y}) $\subseteq$ U^c and therefore (1, 2)*- $M_{m\pi}$ –cl ({x}) $\subseteq$ U^c or x $\in$ U^c, which is a contradiction. Hence case (b) is not possible.

Theorem 3. 29. Let X be any biminimal space. Then the following are equivalent.

1. X is (1, 2)*- $M_{m\pi}$ –R_1 space.
2. For any x, y $\in$ X, one of the following holds:
   i. For U $\in$ (1, 2)*- $M_{m\pi}$ –O(X), x $\in$ U iff y $\in$ V.
   ii. There exists disjoint (1, 2)*- $M_{m\pi}$ –open sets U and V such that x $\in$ U, y $\in$ V.
3. If x, y $\in$ X such that (1, 2)*- $M_{m\pi}$ –cl ({x}) $\neq$ (1, 2)*- $M_{m\pi}$ –cl ({y}), then there exists (1, 2)*- $M_{m\pi}$ –closed sets F_1 and F_2 such that x$\in$ F_1, y $\notin$ F_1, y $\in$ F_2, x $\notin$ F_1 and X = F_1 U F_2.

Proof. 1 => 2: Let x, y $\in$ X. Then (1, 2)*- $M_{m\pi}$ –cl ({x}) = (1, 2)*- $M_{m\pi}$ –cl ({y}) or (1, 2)*- $M_{m\pi}$ –cl ({x}) $\neq$ (1, 2)*- $M_{m\pi}$ –cl ({y}). If (1, 2)*- $M_{m\pi}$ –cl ({x}) = (1, 2)*- $M_{m\pi}$ –cl ({y}) and

$U \in (1, 2)^*$- $M_{m\pi}$ –O(X), then $x \in U \Rightarrow y \in (1, 2)^*$- $M_{m\pi}$ –cl({y}) = (1, 2)*- $M_{m\pi}$ –cl({x}) $\subseteq$ U (as X is (1, 2)*- $M_{m\pi}$ –R_0). If (1, 2)*- $M_{m\pi}$ –cl({x}) $\neq$ (1, 2)*- $M_{m\pi}$ –cl({y}), then there exists U, V $\in$ (1, 2)*- $M_{m\pi}$ –O(X) such that x $\in$ (1, 2)*- $M_{m\pi}$ –cl({x}) $\subseteq$ U, y $\in$ (1, 2)*- $M_{m\pi}$ –cl({y}) $\subseteq$ V and U ∩ V = φ.

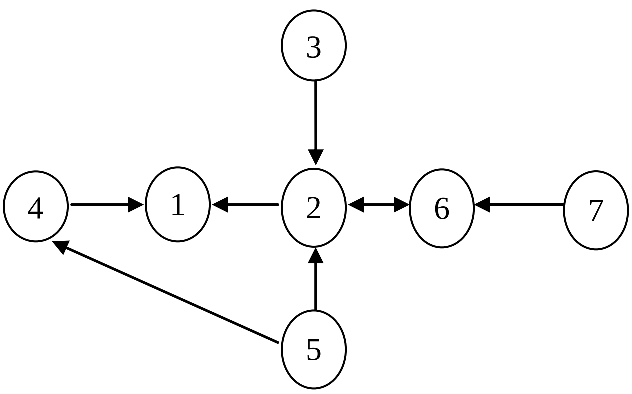
2 => 3: Let x, y $\in$ X such that (1, 2)*- $M_{m\pi}$ –cl({x}) $\neq$ (1, 2)*- $M_{m\pi}$ –cl({y}). Then x $\notin$ (1, 2)*- $M_{m\pi}$ –cl({y}), so that there exists G $\in$ (1, 2)*- $M_{m\pi}$ –O(X) such that x $\in$ G and y $\notin$ G. Thus by [2], there exists disjoint (1, 2)*- $M_{m\pi}$ –open sets U and V such that x $\in$ U, y $\in$ V. Put $F_1 = X \setminus V$ and $F_2 = X \setminus U$. Then $F_1, F_2 \in$ (1, 2)*- $M_{m\pi}$ –C(X), x $\in$ F_1, y $\notin$ F_1, y $\in$ F_2, x $\notin$ F_2 and X = F_1 U F_2.

3 => 1: Let U $\in$ (1, 2)*- $M_{m\pi}$ –O(X) and x $\in$ U. Then (1, 2)*- $M_{m\pi}$ -cl ({x}) $\subseteq$ U. In fact, otherwise there exists y $\in$ (1, 2)*- $M_{m\pi}$ – cl ({x}) ∩ (X \ U). Then (1, 2)*- $M_{m\pi}$ –cl ({x}) $\neq$ (1, 2)*- $M_{m\pi}$ –cl ({y}) (as x $\notin$ (1, 2)*- $M_{m\pi}$ –cl ({y})) and so by [3], there exists $F_1, F_2 \in$ (1, 2)*- $M_{m\pi}$ –C(X) such that x $\in$ F_1, y $\notin$ F_1, y $\in$ F_2, x $\notin$ F_2 and X = F_1 U F_2. Then y $\in$ $F_2 \setminus F_1 = X \setminus F_1$ and x $\notin$ $X \setminus F_1$, where $X \setminus F_1 \in$ (1, 2)*- $M_{m\pi}$ –O(X), which is a contradiction to the fact that y $\in$ (1, 2)*- $M_{m\pi}$ -cl ({x}). Hence (1, 2)*- $M_{m\pi}$ -cl ({x}) $\subseteq$ U. Thus X is (1, 2)*- $M_{m\pi}$ –R_0. To show X to be (1, 2)*- $M_{m\pi}$ – R_1 assume that a, b $\in$ X with (1, 2)*- $M_{m\pi}$ –cl ({a}) $\neq$ (1, 2)*- $M_{m\pi}$ –cl ({b}). Then as above, there exists $P_1, P_2 \in$ (1, 2)*- $M_{m\pi}$ –C(X) such that a $\in$ P_1, b $\notin$ P_1, b $\in$ P_2, a $\notin$ P_2 and X = P_1 U P_2. Thus a $\in$ $P_1 \setminus P_2 \in$ (1, 2)*- $M_{m\pi}$ –O(X), b $\in$ $P_2 \setminus P_1 \in$ (1, 2)*- $M_{m\pi}$ –O(X). So (1, 2)*- $M_{m\pi}$ -cl ({a}) $\subseteq$ $P_1 \setminus P_2$. (1, 2)*- $M_{m\pi}$ -cl ({b}) $\subseteq$ $P_2 \setminus P_1$. Thus X is (1, 2)*- $M_{m\pi}$ –R_1 space.

Remark 3. 30. From the above theorems and examples we have the following implications.

1. (1, 2)*- $M_{m\pi}$ –T_0. 2. (1, 2)*- $M_{m\pi}$ –T_1. 3. (1, 2)*- $M_{m\pi}$ -T_2. 4. (1, 2)*- $M_{m\pi}$ –C_0

5. (1, 2)*- $M_{m\pi}$ –C_1 6. (1, 2)*- $M_{m\pi}$ –R_0 7. (1, 2)*- $M_{m\pi}$ –R_1.



Definition 3. 31. A space X is said to be (1, 2)*- $M_{m\pi}$ -regular for each (1, 2)*- $M_{m\pi}$ -closed set F and each point $x \notin F$ there exist disjoint (1, 2)*- $M_{m\pi}$ -open sets U and V such that x Є U and F $\subset$ V.

Theorem 3. 32. An (1, 2)*- $M_{m\pi}$ -T_0-space is (1, 2)*- $M_{m\pi}$ -T_2-space if it is (1, 2)*- $M_{m\pi}$ -regular.

Proof. Let X be (1, 2)*- $M_{m\pi}$ -T_0-space and (1, 2)*- $M_{m\pi}$ -regular. If x, y$\in$X, x $\neq$y, there exists U Є (1, 2)*-$M_{m\pi}$-O(X) such that U contains one of x and y, say x but not y. Then X\U is (1, 2)*- $M_{m\pi}$ -closed and x $\notin$ X \ U. Since X is (1, 2)*- $M_{m\pi}$ -regular, there exist disjoint (1, 2)*- $M_{m\pi}$ -open sets V_1 and V_2 such that x$\in V_1$ and X \ U $\subset$ V_2. Thus x$\in V_1$ and y$\in V_2$, $V_1 \cap V_2 = \varphi$. Hence X is (1, 2)*- $M_{m\pi}$ -T_2-space.

4. (1, 2)*- $M_{m\pi}$ –NEIGHBOURHOOD AND (1, 2)*- $M_{m\pi}$ – ACCUMULATION POINTS

Definition 4. 1. A subset N of X is said to be (1, 2)*- $M_{m\pi}$ –neighbourhood of a point x Є X if there exist (1, 2)*- $M_{m\pi}$ -open set G of X such that x Є G $\subseteq$ N.

Example 4. 2. Let X = {a, b, c}, τ_1 = {φ, X, {a}, {b}} and τ_2 = {φ, X, {a, c}, {b, c}}. Here {φ, X, {a}, {b}, {a, c}, {b, c}} are (1, 2)*- $M_{m\pi}$ –open sets in X. Then, {b}, {a, b}, {b, c} and X are (1, 2)*- $M_{m\pi}$ –neighbourhood of {b}.

Theorem 4. 3. Let X be a biminimal space. If N$\subseteq$M and N is (1, 2)*- $M_{m\pi}$ –neighbourhood of a point x, then M is (1, 2)*- $M_{m\pi}$ –neighbourhood of a point x.

Proof. Suppose that N$\subseteq$M and N is (1, 2)*- $M_{m\pi}$ –neighbourhood of a point x. Thus there exists (1, 2)*- $M_{m\pi}$ –open set G of X such that x Є G $\subseteq$N. By assumption, we have N$\subseteq$M. The theorem is now complete.

Theorem 4. 4. Let X be a biminimal space, G be any subset of X and x $\in$X. G is (1, 2)*- $M_{m\pi}$ -open set of X if and only if G is (1, 2)*- $M_{m\pi}$ –neighbourhood of x for any x $\in$G.

Proof. Let X be a biminimal space, G be any subset of X and x $\in$X.

Suppose that G is (1, 2)*- $M_{m\pi}$ –open set of X.

Case 1. If G = φ, it is clear.

Case 2. If G $\neq$ φ, let x $\in$G. Since G is (1, 2)*- $M_{m\pi}$ –open and G $\subseteq$G, G is (1, 2)*- $M_{m\pi}$ –neighbourhood of x

Conversely, suppose that G is (1, 2)*- $M_{m\pi}$ –neighbourhood of x for any x Є G. Now, we would like to show that G is (1, 2)*- $M_{m\pi}$ –open. Since x Є G and G is (1, 2)*- $M_{m\pi}$ –neighbourhood of x, there exists (1, 2)*- $M_{m\pi}$ –open set U_x such that x Є $U_x \subseteq$G and so {x} $\subseteq$ $U_x \subseteq$G. It follows that, G = $\bigcup_{x \in G}\{x\} \subseteq \bigcup_{x \in G} U_x \subseteq \bigcup_{x \in G} G$ = G, G = $\bigcup_{x \in G} U_x$

Since U_x is (1, 2)*- $M_{m\pi}$ –open for any x ∈ G and by Theorem 3. 7[8], we have G is (1, 2)*- $M_{m\pi}$ –open set of X.

Theorem 4. 5. For a space X, the following statements are equivalent.

1. X is (1, 2)*- $M_{m\pi}$-T_2.
2. If x ∈ X, then for each y ≠ x, there is an (1, 2)*- $M_{m\pi}$-neighbourhood N(x) of x, such that y ∉ (1, 2)*- $M_{m\pi}$-cl (N(x)).
3. For each x ∈ {(1, 2)*- $M_{m\pi}$-cl (N): N is an (1, 2)*- $M_{m\pi}$-neighbourhood of x} = {x}.

Proof. 1 => 2: Let x ∈ X. If y ∈ X is such that y ≠ x, there exist disjoint (1, 2)*- $M_{m\pi}$-open sets U, V such that x ∈ U and y ∈ V. Then x ∈ U ⊆ X – V which implies that X – V is an (1, 2)*- $M_{m\pi}$-neighbourhood of x. Also X –V is (1, 2)*- $M_{m\pi}$-closed and y ∉ X – V. Let N(x) = X – V. Then y ∉ (1, 2)*- $M_{m\pi}$-cl (N(x)).

2 => 3: Obvious.

3 => 1: Let x, y ∈ X, x ≠ y. By hypothesis, there is atleast an (1, 2)*- $M_{m\pi}$-neighbourhood N of x such that y ∉ (1, 2)*- $M_{m\pi}$-cl (N). We have x ∉ X - (1, 2)*- $M_{m\pi}$-cl (N) is (1, 2)*- $M_{m\pi}$-open. Since N is an (1, 2)*- $M_{m\pi}$-neighbourhood of x, there exists U ∈ (1, 2)*- $M_{m\pi}$-O(X) such that x ∈ U ⊆ N and U ∩(X - (1, 2)*- $M_{m\pi}$-cl (N)) = φ. Hence X is (1, 2)*- $M_{m\pi}$-T_2.

Definition 4. 6. A point x of X is called a (1, 2)*- $M_{m\pi}$ –accumulation point of a subset A of X if G ∩ (A– {x}) ≠ φ for any (1, 2)*- $M_{m\pi}$ –open set G in X such that x ∈ G.

We denote the set of all (1, 2)*- $M_{m\pi}$ –accumulation point of A by (1, 2)*- $M_{m\pi}$ –acc (A).

Example 4. 7. In Example 4. 2, {3} is (1, 2)*- $M_{m\pi}$ –accumulation point of X and (1, 2)*- $M_{m\pi}$ –acc(X) = {3}.

Lemma 4. 8. Let X is a biminimal space and A, B be a subset of X. If A ⊆ B, then (1, 2)*- $M_{m\pi}$ –acc (A) ⊆ (1, 2)*- $M_{m\pi}$ –acc (B).

Proof. Let A ⊆ B and x ∈ (1, 2)*- $M_{m\pi}$ –acc (A). Then for any (1, 2)*- $M_{m\pi}$ –open set G in X such that x ∈ G, G ∩ (A – {x}) ≠ φ. Since A- {x} ⊆ B-{x} and so φ ≠ G ∩ (A – {x}) ⊆ G ∩ (B – {x}). Hence x ∈ (1, 2)*- $M_{m\pi}$ –acc (B).

Theorem 4. 9. Let X be a biminimal space and A, B be a subset of X. Then (1, 2)*- $M_{m\pi}$ –acc (A ∩ B) ⊆ (1, 2)*- $M_{m\pi}$ –acc (A) ∩ (1, 2)*- $M_{m\pi}$ –acc (B).

Proof. Let A∩B ⊆ A, A∩B ⊆ B and Lemma 4. 8, we obtain that (1, 2)*- $M_{m\pi}$ –acc (A ∩ B) ⊆ (1, 2)*- $M_{m\pi}$ –acc (A) and (1, 2)*- $M_{m\pi}$ –acc (A ∩ B) ⊆ (1, 2)*- $M_{m\pi}$ –acc (B). Therefore, (1, 2)*- $M_{m\pi}$ –acc (A ∩ B) ⊆ (1, 2)*- $M_{m\pi}$ –acc (A) ∩ (1, 2)*- $M_{m\pi}$ –acc (B).

Theorem 4. 10. Let X be a biminimal space and A, B be a subset of X. A is (1, 2)*- $M_{m\pi}$ –closed set of X if and only if (1, 2)*- $M_{m\pi}$ –acc (A) ⊆ A.

Proof. Let X be a biminimal space and A ⊆ X.

Assume that A is (1, 2)*- $M_{m\pi}$ –closed set of X. Suppose that (1, 2)*- $M_{m\pi}$ –acc (A) ⊈ A. Thus there exists x ∈ (1, 2)*- $M_{m\pi}$ –acc (A), but x ∉ A. Since x ∈ (1, 2)*- $M_{m\pi}$ –acc (A), G ∩ (A – {x}) ≠ φ for any (1, 2)*- $M_{m\pi}$ –open set G in X such that x ∈ G. Since x ∉ A, G ∩ A = G ∩ (A – {x}) ≠ φ for any (1, 2)*- $M_{m\pi}$ –open set G in X such that x ∈ G. By assumption, we get X – A is (1, 2)*- $M_{m\pi}$ –open and x ∈ X –A. It follows that (X – A) ∩ A ≠ φ, this is contradiction. Therefore, (1, 2)*- $M_{m\pi}$ –acc (A) ⊆ A.

Conversely, Assume that (1, 2)*- $M_{m\pi}$ -acc (A) ⊆ A. Next we would like to show that A is (1, 2)*- $M_{m\pi}$ -closed set of X, i. e., we must to show that X – A is (1, 2)*- $M_{m\pi}$ –open set of X.

Case 1. If X – A = φ, then A is (1, 2)*- $M_{m\pi}$ –closed set of X.

Case 2. If X – A ≠ φ. Let x ∈ X – A. Thus x ∉ A. Since (1, 2)*- $M_{m\pi}$ -acc (A) ⊆ A, x ∉ (1, 2)*- $M_{m\pi}$ -acc (A). Thus there exists (1, 2)*- $M_{m\pi}$ –open set G in X such that x ∈ G and G ∩ (A – {x}) = φ. Since x ∉ A, G ∩ A = G ∩ (A – {x}) = φ and we also have x ∈ G ⊆ (X – A). Thus X –A is (1, 2)*- $M_{m\pi}$ –neighbourhood of x. By Theorem 4. 4, we can imply that X –A is (1, 2)*- $M_{m\pi}$ – open set of X. Consequently A is (1, 2)*- $M_{m\pi}$ –closed set of X.

REFERENCES

1. **Ashish Kar and Paritosh Bhattacharyya,** Some weak separation axioms, Bull. Cal. Math. Soc. 82(1990), 415-422.
2. **S. Athisaya Pomona and M. Lellis Thivagar,** Another form of separation Axioms, Methods of Functional Analysis and Topology, Vol. 13(2007), no.4, pp. 380-385.
3. **M. Caldas and S. Jafari,** On some low separation axioms in topological spaces, Houston Journal of Math., **29** (2003), 93–104.
4. **M. Caldas, S. Jafari, S. A. Ponmani and M. Lellis Thivagar,** On some low separation axioms in bitopological spaces, Bol. Soc. Paran. Mat.(3s) v.24 1-2(2006);69-78.

5. **A. S. Davis**, Indexed systems of neighborhoods for general topological spaces, Amer. Math. Soc., **68** (1961), 886–893.
6. **S.N. Maheswari and R. Prasad,** Some new separation axioms, Ann. Soc. Sci. Bruxelles, 89(1975), 395-402.
7. **S. N. Maheswari and R. Prasad,** On R_0-spaces, Portugal Math., 34(1975), 213-217.
8. **K. Mohana and I. Arockiarani,** (1, 2)*-$M_{m\pi}$-Closed sets In Biminimal Spaces (Communicated).
9. **M. G. Murdeshwar and S. A. Naimpally**, *R_1-topological spaces*, Canad. Math. Bull., 9 (1996), 521–523.
10. **S. A. Naimpally,** *On R_0-topological spaces,* Ann. Univ. Sci. Budapest. E¨otv¨os Sect. Math. 10 (1976), 53–54.
11. **T. Noiri,** 11th meeting on topological spaces and its Applications, Fukuoka University Seminar House, 2006, 1-9.
12. V. **Popa and T. Noiri,** On M-continuous functions, Anal. Univ "Dunarea de Jos " Galati, Ser. Mat. Fiz. Mec. Tecor. (2), 18(23) (2000), 31-41.
13. **O. Ravi, R. G. Balamurugan and M. Balakrishnan**, On biminimal quotient mappings, International Journal of Advances in Pure and Applied Mathematics, 1(2) (2011), 96-112.
14. **O. Ravi, R. G. Balamurugan and M. Krishnamoorthy**, Decompositions of $M^{(1, 2)*}$-continuity and Complete $M^{(1, 2)*}$-continuity In Biminimal Spaces, International Journal of Mathematical Archive, 2(11) (2011), 2299-2307.
15. **N. A. Shanin**, On separation in topological spaces, Dokl. Akad. Nauk SSSR, 38 (1943), 110–113.
16. **C.-T. Yang,** "On paracompact spaces," Proc. Amer. Math. Soc., vol. 5, pp. 185–189, 1954.