The Role MCDM or MCDA Approaches in Criteria Decisions

Naser Al-Dori, Ferank Kertin, Fabio Casatio

Department of Mathematical, University of North Maraca, Maraca

Email: fabi.casatio@gmail.com

Abstract: The difficulty of the problem originates from the presence of more than one criterion. There is no longer a unique optimal solution to an MCDM problem that can be obtained without incorporating preference information. The conventional methods for selection are inadequate for dealing with the imprecise or vague nature of linguistic assessment. To overcome this difficulty, multicriteria decision-making methods are proposed. The aim of this study is to use analytic hierarchy process (AHP) and the technique for order preference by similarity to ideal solution (TOPSIS) methods for Multi-criteria evaluation decision. It is unusual that the cheapest car is the most comfortable and the safest one. In portfolio management, we are interested in getting high returns but at the same time reducing our risks. This paper presents a comparison of MCDM and MCDA in mine case study.

Keywords: Multiple-criteria decision, AHP, Criteria Decisions, Multi evaluation decision, MCDM.

1. Introduction

Multiple-criteria decision-making or multiple-criteria decision analysis (MCDA) is a sub-discipline of operations research that explicitly considers multiple criteria in decision-making environments. Whether in our daily lives or in professional settings, there are typically multiple conflicting criteria that need to be evaluated in making decisions. Cost or price is usually one of the main criteria [1, 2, 3]. Some measure of quality is typically another criterion that is in conflict with the cost. In purchasing a car, cost, comfort, safety, and fuel economy may be some of the main criteria we consider. It is unusual that the cheapest car is the most comfortable and the safest one. In portfolio management, we are interested in getting high returns but at the same time reducing our risks [4]. Again, the stocks that have the potential of bringing high returns typically also carry high risks of losing money. In a service industry, customer satisfaction and the cost of providing service are two conflicting criteria that would be useful to consider [5, 6].

In our daily lives, we usually weigh multiple criteria implicitly and we may be comfortable with the consequences of such decisions that are made based on only intuition. On the other hand, when stakes are high, it is important to properly structure the problem and explicitly evaluate multiple criteria. In making the decision of whether to build a nuclear power plant or not, and where to build it, there are not only very complex issues involving multiple criteria, but there are also multiple parties who are deeply affected from the consequences [7, 8].

Structuring complex problems well and considering multiple criteria explicitly leads to more informed and better decisions. There have been important advances in this field since the start of the modern multiple-criteria decision-making discipline in the early 1960s. A variety of approaches and methods, many implemented by specialized decision-making software [9, 10]] have been developed for their application in an array of disciplines, ranging from politics and business to the environment and energy [11]. MCDM or MCDA are well-known acronyms for multiple-criteria decision-making and multiple-criteria decision analysis. Stanley Zionts wrote an article in 1979 titled: "MCDM – If not a Roman Numeral, then What"?

MCDM is concerned with structuring and solving decision and planning problems involving multiple criteria. The purpose is to support decision makers facing such problems [12, 13, 14]. Typically, there does not exist a unique optimal solution for such problems and it is necessary to use decision maker's preferences to differentiate between solutions. "Solving" can be interpreted in different ways. It could correspond to choosing the "best" alternative from a set of available alternatives (where "best" can be interpreted as "the most preferred alternative" of a decision maker). Another interpretation of "solving" could be choosing a small set of good alternatives, or grouping alternatives into different preference sets. An extreme interpretation could be to find all "efficient" or "nondominated" alternatives (which we will define shortly [15, 16, 17].

The difficulty of the problem originates from the presence of more than one criterion. There is no longer a unique optimal solution to an MCDM problem that can be obtained without incorporating preference information. The concept of an optimal solution is often replaced by the set of nondominated solutions [18, 19]. A nondominated solution has the property that it is not possible to move away from it to any other solution without sacrificing in at least one criterion (figure 1). Therefore, it makes sense for the decision maker to choose a solution from the nondominated set. Otherwise, he could do better in terms of some or all of the criteria, and not do worse in any of them. Generally, however, the set of nondominated solutions is too large to be presented to the decision maker for his final choice. Hence we need tools that help the decision maker focus on his preferred solutions (or alternatives). Normally one has to "tradeoff" certain criteria for others [20, 21, 22].

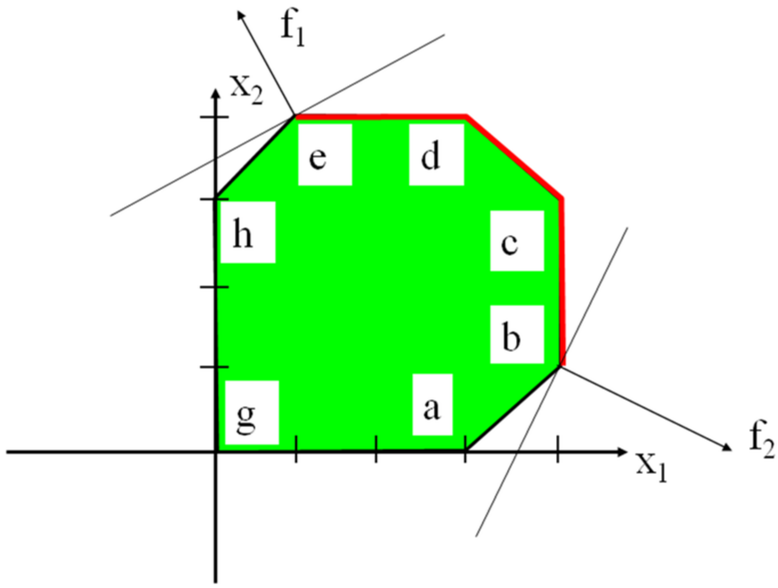

Figure1. Demonstration of the decision space

2. TOPSIS

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a multi-criteria decision analysis method, which was originally developed by Hwang and Yoon in 1981[23] with further developments by Yoon in 1987 [24] and Hwang, Lai and Liu in 1993. TOPSIS is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution. It is a method of compensatory aggregation that compares a set of alternatives by identifying weights for each criterion, normalising scores for each criterion and calculating the geometric distance between each alternative and the ideal alternative, which is the best score in each criterion. An assumption of TOPSIS is that the criteria are monotonically increasing or decreasing. Normalisation is usually required as the parameters or criteria are often of incongruous dimensions in multi-criteria problems. Compensatory methods such as TOPSIS allow trade-offs between criteria, where a poor result in one criterion can be negated by a good result in another criterion. This provides a more realistic form of modelling than non-compensatory methods, which include or exclude alternative solutions based on hard cut-offs [25, 26].

Step 1: The value of fuzzy synthetic extent with respect to the i^{th} object is defined as

$$S_i = \sum_{j=1}^{m} M_{gi}^{m} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{m}\right]^{-1} \qquad (1)$$

To determine $\sum_{j=1}^{m} M_{gi}^{m}$, perform the fuzzy addition operation of m extent analysis values for a particular matrix such that (Chang, 1996; Zare et al, 2009):

$$\sum_{j=1}^{m} M_{gi}^{m} = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right) \quad (2)$$

And to determine $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{m}\right]^{-1}$, perform the fuzzy edition operation of m extent analysis values for a particular matrix such that (Chang, 1996; Zare et al, 2009):

$$\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{m} = \left(\sum_{i=1}^{n} l_j, \sum_{i=1}^{n} m_j, \sum_{i=1}^{n} u_j\right) \quad (3)$$

And then obtain the opposite of the vector in Eq. (4) such that (Chang, 1996; Zare et al, 2009):

$$V(M_2 \geq M_1) = \sup\left[\min(\mu_{M_1}(x), \mu_{M_2}(y))\right] \quad (4)$$

Where d is the ordinate of the highest junction point D between μ_{M_1} and μ_{M_2} (see Figure 2).

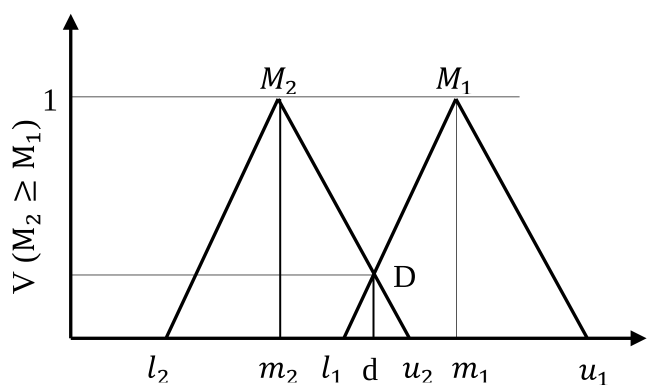

Figure2. The junction between M_1 and M_2

There are different classifications of MCDM problems and methods. A major distinction between MCDM problems is based on whether the solutions are explicitly or implicitly defined. Multiple-criteria evaluation problems: These problems consist of a finite number of alternatives, explicitly known in the beginning of the solution process. Each alternative is represented by its performance in multiple criteria. The problem may be defined as finding the best alternative for a decision maker (DM), or finding a set of good alternatives. One may also be interested in "sorting" or "classifying" alternatives. Sorting refers to placing alternatives in a set of preference-ordered classes (such as assigning credit-ratings to countries), and classifying refers to assigning alternatives to non-ordered sets (such as diagnosing patients based on their symptoms). Some of the MCDM methods in this category have been studied in a comparative manner in the book by Triantaphyllou on this subject, 2000 [27].

Multiple-criteria design problems (multiple objective mathematical programming problems): In these problems, the alternatives are not explicitly known. An alternative (solution) can be found by solving a mathematical model. The number of alternatives is either infinite and not countable (when some variables are continuous) or typically very large if countable (when all variables are discrete. Whether it is an evaluation problem or a design problem, preference information of DMs is required in order to differentiate between solutions. The solution methods for MCDM problems are commonly classified based on the timing of preference information obtained from the DM.

There are methods that require the DM's preference information at the start of the process, transforming the problem into essentially a single criterion problem. These methods are said to operate by "prior articulation of preferences." Methods based on estimating a value function or using the concept of "outranking relations," analytical hierarchy process, and some decision rule-based methods try to solve multiple criteria evaluation problems utilizing prior articulation of preferences. Similarly, there are methods developed to solve multiple-criteria design problems using prior articulation of preferences by constructing a value function. Perhaps the most well-known of these methods is goal programming. Once the value function is constructed, the resulting single objective mathematical program is solved to obtain a preferred solution. Some methods require preference information from the DM throughout the solution process. These are referred to as interactive methods or methods that require "progressive articulation of preferences." Multiple-criteria design problems typically require the solution of a series of mathematical programming models in order to reveal implicitly defined solutions. For these problems, a representation or approximation of "efficient solutions" may also be of interest. When the mathematical programming models contain integer variables, the design problems become harder to solve.

3. Analytic hierarchy process (AHP)

The analytic hierarchy process (AHP) is a structured technique for organizing and analyzing complex decisions, based on mathematics and psychology. It was developed by Thomas L. Saaty in the 1970s and has been extensively studied and refined since then.

It has particular application in group decision making [21] and is used around the world in a wide variety of decision situations, in fields such as government, business, industry, healthcare, shipbuilding [22] and education. Rather than prescribing a "correct" decision, the AHP helps decision makers find one that best suits their goal and their understanding of the problem. It provides a comprehensive and rational framework for structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions. Users of the AHP first decompose their decision problem into a hierarchy of more easily comprehended sub-problems, each of which can be analyzed independently. The elements of the hierarchy can relate to any aspect of the decision problem-tangible or intangible, carefully measured or roughly estimated, well or poorly understood-anything at all that applies to the decision at hand.

Once the hierarchy is built, the decision makers systematically evaluate its various elements by comparing them to one another two at a time, with respect to their impact on an element above them in the hierarchy. In making the comparisons, the decision makers can use concrete data about the elements, but they typically use their judgments about the elements' relative meaning and importance. It is the essence of the AHP that human judgments, and not just the underlying information, can be used in performing the evaluations [20]. The AHP converts these evaluations to numerical values that can be processed and compared over the entire range of the problem. A numerical weight or priority is derived for each element of the hierarchy, allowing diverse and often incommensurable elements to be compared to one another in a rational and consistent way. This capability distinguishes the AHP from other decision making techniques. In the final step of the process, numerical priorities are calculated for each of the decision alternatives (figure 3). These numbers represent the alternatives' relative ability to achieve the decision goal, so they allow a straightforward consideration of the various courses of action.

Where all the $M_{g_i}^{j}$ (j=1, 2,…, m) are triangular fuzzy numbers. The steps of Chang's extent analysis can be given as in the following:

Step 1: The value of fuzzy synthetic extent with respect to the ith object is defined as

$$S_i = \sum_{j=1}^{m} M_{g_i}^{j} \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^{j} \right]^{-1} \qquad (5)$$

To obtain $\sum_{j=1}^{m} M_{g_i}^{j}$, perform the fuzzy addition operation of m extent analysis values for a particular matrix such that

$$\sum_{j=1}^{m} M_{g_i}^{j} \left(\sum_{j=1}^{m} l_j , \sum_{j=1}^{m} m_j , \sum_{j=1}^{m} u_j \right) \qquad (6)$$

And to obtain $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^{j}\right]^{-1}$, perform the fuzzy edition operation of m extent analysis values for a particular matrix such that

Step 2: The degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ is defined as

$$V(M_2 \geq M_1) = \sup\left\lfloor \min(\mu_{M_1}(x), \mu_{M_2}(y)) \right\rfloor \quad (7)$$

And can be equivalently expressed as follows:

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M_2}(d)$$
$$= \begin{cases} 1 & if\ m_2 \geq m_1, \\ 0 & if\ l_2 \geq u_2, \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} & otherwise. \end{cases} \quad (8)$$

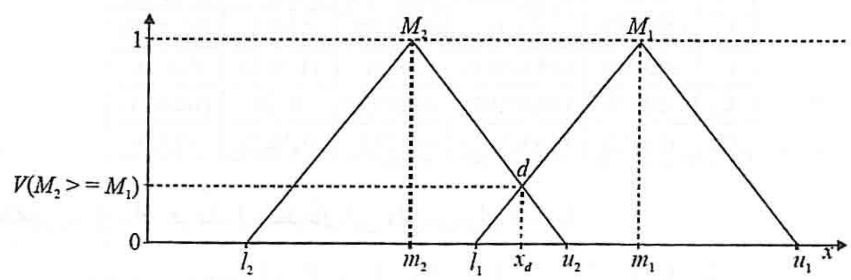

Figure3. the intersection between m_1 and m_2

As can be seen in the material that follows, using the AHP involves the mathematical synthesis of numerous judgments about the decision problem at hand. It is not uncommon for these judgments to number in the dozens or even the hundreds. While the math can be done by hand or with a calculator, it is far more common to use one of several computerized methods for entering and synthesizing the judgments. The simplest of these involve standard spreadsheet software, while the most complex use custom software, often augmented by special devices for acquiring the judgments of decision makers gathered in a meeting room.

The procedure for using the AHP can be summarized as:

- Model the problem as a hierarchy containing the decision goal, the alternatives for reaching it, and the criteria for evaluating the alternatives.
- Establish priorities among the elements of the hierarchy by making a series of judgments based on pairwise comparisons of the elements. For example, when comparing potential purchases of commercial real estate, the investors might say they prefer location over price and price over timing.
- Synthesize these judgments to yield a set of overall priorities for the hierarchy. This would combine the investors' judgments about location, price and timing for properties A, B, C, and D into overall priorities for each property.
- Check the consistency of the judgments.
- Come to a final decision based on the results of this process.

4. Comparison

The AHP is included in most operations research and management science textbooks, and is taught in numerous universities; it is used extensively in organizations that have carefully investigated its theoretical underpinnings [5]. While the general consensus is that it is both technically valid and practically useful, the method does have its critics [8]. Most of the criticisms involve a phenomenon called rank reversal, discussed in the following section.

The decision space corresponds to the set of possible decisions that are available to us. The criteria values will be consequences of the decisions we make. Hence, we can define a corresponding problem in the decision space. For example, in designing a product, we decide on the design parameters (decision variables) each of which have an impact on the performance measures (criteria) with which we evaluate our product.

Mathematically, a multiple-criteria design problem can be represented in the decision space as follows:

"max" $q = f(x) = (f_1(x),...,f_k(x))$

subject to $q \in Q = \{f(x) : x \in X, X \subseteq R^n\}$,

where X is the feasible set and x is the decision variable vector of size n.

A well-developed special case is obtained when X is a polyhedron defined by linear inequalities and equalities. If all the objective functions are linear in terms of the decision variables, this variation leads to multiple objective linear programming (MOLP), an important subclass of MCDM problems.

There are several definitions that are central in MCDM. Two closely related definitions are those of nondominance (defined based on the criterion space representation) and efficiency (defined based on the decision variable representation).

Definition 1. $q^* \in Q$ is nondominated if there does not exist another $q \in Q$ such that $q \geq q^*$ and $q \neq q^*$.

Roughly speaking, a solution is nondominated so long as it is not inferior to any other available solution in all the considered criteria.

Definition 2. $x^* \in X$ is efficient if there does not exist another $x \in X$ such that $f(x) \geq f(x^*)$ and $f(x) \neq f(x^*)$.

If an MCDM problem represents a decision situation well, then the most preferred solution of a DM has to be an efficient solution in the decision space, and its image is a nondominated point in the criterion space. Following definitions are also important.

Definition 3. $q^* \in Q$ is weakly nondominated if there does not exist another $q \in Q$ such that $q > q^*$.

Definition 4. $x^* \in X$ is weakly efficient if there does not exist another $x \in X$ such that $f(x) > f(x^*)$.

Weakly nondominated points include all nondominated points and some special dominated points. The importance of these special dominated points comes from the fact that they commonly appear in practice and special care is necessary to distinguish them from nondominated points. If, for example, we maximize a single objective, we may end up with a weakly nondominated point that is dominated. The dominated points of the weakly nondominated set are located either on vertical or horizontal planes (hyperplanes) in the criterion space.

Ideal point: (in criterion space) represents the best (the maximum for maximization problems and the minimum for minimization problems) of each objective function, and typically corresponds to an infeasible solution.

Nadir point: (in criterion space) represents the worst (the minimum for maximization problems and the maximum for minimization problems) of each objective function among the points in the nondominated set, and is typically a dominated point.

The ideal point and the nadir point are useful to the DM to get the "feel" of the range of solutions (although it is not straightforward to find the nadir point for design problems having more than two criteria).

5. Conclusion

By using AHP and TOPSIS, uncertainty and vagueness from subjective perception and the experiences of decision-maker can be effectively represented and reached to a more effective decision. In this study plant Species selection with AHP and TOPSIS method has been proposed. Although two methods have the same objective of selecting the best plant Species for the mine reclamation, they have differences. In TOPSIS decision-makers used the linguistic variables to assess the importance of the criteria and to evaluate the each alternative with respect to each criterion. These linguistic variables converted into triangular fuzzy numbers and decision matrix was formed. Then normalized decision matrix and weighted.normalized decision matrix were formed. After FPIS and FNIS were defined, distance of each alternative to FPIS and FNIS were calculated. And then the closeness coefficient of each alternative was calculated separately.

References

1. EK. Aleksandrovskaya, and VP. Urakhchin, Prediction of the displacements of concrete gravity dams on rock foundations, Power Technology and Engineering (formerly Hydrotechnical Construction) 8, 419-427 (1974).
2. CR. Allen, and LS. Cluff, Active faults in dam foundations: an update, Proc. 12th World Conf. on Earthquake Engineering, Auckland, New Zealand (2000).
3. M. Wieland, R.P. Brenner, and P. Sommer, Earthquake resilience of large concrete dams: Damage, repair, and strengthening concepts, Trans. 21st Int. Congress on Large Dams, Montreal 131-150 (2003).
4. J. Mata, and E. Portela, Application of neural networks to dam safety control, 5the International Conference on Dam Engineering, Lisbon, Portugal (2007).
5. M. Mokhtari, H. Alinejad-Rokny, H. Jalalifar, Selection of the Best Well Control System by Using Fuzzy Multiple-Attribute Decision-Making Methods, Journal of Applied Statistics, 41(5): 1105-1121 (2014).
6. M. Wieland, and P. Brenner, Potentially active faults in the foundations of large dams part I: Vulnerability of dams TO seismic movements in dam foundation, Proc. 14th World Conf. On Earthquake Engineering, Beijing, China, (2008).
7. N. Seyedaghaee, S. Rahati, H. Alinejad-Rokny and F. Rouhi, An Optimized Model for the University Strategic Planning, International Journal of Basic Sciences & Applied Research, 2(5): 500-505 (2013).
8. YS. Kim, and B. Kim, Prediction of relative crest settlement of concrete-faced rockfill dams analyzed using an artificial neural network model, Computers and Geotechnics 35, 313-322 (2008).
9. J. Mata, Interpretation of concrete dam behavior with artificial neural network and multiple linear regression models, Engineering Structures 33, 903-910 (2011).
10. H. Kamali-Bandpey, H. Alinejad-Rokny, H. Khanbabapour, F. Rashidinejad, Optimization of Firing System Utilizing Fuzzy Madm Method-Case study: Gravel Mine Project in Gotvand Olya Dam-Iran, Australian Journal of Basic & Applied Sciences, 5(12): 1089-1097 (2011).
11. S Adnani, F Sereshki, H Alinejad-Rokny, H Kamali-Bandpey, Selection of Temporary Ventilation System for Long Tunnels by Fuzzy Multi Attributes Decision Making Technique (Fuzzy-Madm), Case Study: Karaj Water Conveyance Tunnel (Part Et-K′) in Iran, American Journal of Scientific Research, 29: 83-91, (2011).

12. F. Chan, Integration of Expert System with Analysis Hierarchy Process for the Design of Material Handling Selection System, The Journal of Material Processing Technology 32, 137-145 (2001).

13. O. Kulak, Multi-Attribute Material Handling Equipment Selection Using Information Axiom, the Third International Conference on Axiomatic Design, Seoul 24-24 (2004).

14. S. Chakraborthy, Design of Material Handling Equipment Selection Model Using Analytic Hierarchy Process, International Journal of Advanced Manufacturing 28, 1237-1245 (2006).

15. A. Reza Fazli, M. Eghbali Asli, H. Alinejad-Rokny, Improving Security of Audio Watermarking in Image using Selector Keys, Research Journal of Applied Sciences, Engineering and Technology, 4(11): 1545-1549 (2012).

16. A. Aghajani, and M. Akbarpour, Optimizing Loading System of Gol-e-Gohar Iron Ore Mine of Iran by Genetic Algorithm, Iron Ore Conference, Australia 55-63 (2007).

17. AT. Gumus, Evaluation of Hazardous Waste Transportation Firms by Using a Two Step Fuzzy-AHP and TOPSIS Methodology, Expert Systems with Applications 36, 4067-4074 (2009).

18. I. Alavi, A. Akbari, H. Alinejad-Rokny, Plant Type Selection for Reclamation of Sarcheshmeh Copper Mine by Fuzzy-TOPSIS Method, Advanced Engineering Technology and Application, 1(1): 8-13 (2012).

19. A. Bascetin, An Application of the Analytic Hierarchy Process in Equipment Selection at Orhaneli Open Pit Coal Mine, Mining Technology (Trans. Inst. Min. Metall. A) 113, A192-A199 (2004).

20. G. Enea, and G. Galante, An Integrated Approach to the Facilities and Material Handling System Design, International Journal of Production Research 40, 4007-4017 (2002).

21. N. Celebi, An Equipment Selection and Cost Analysis System for Open Pit Coal Mines, International Journal of Mining, Reclamation and Environment 12, 181-187 (1998).

22. D. Denby, and D. Schofield, Application of Expert Systems in Equipment Selection for Surface Design, International Journal of Mining, Reclamation and Environment 4, 165-171 (1990).

23. S. Bandopadhyay, and P. Venkatasubramanian, Expert Systems as Decision Aid in Surface Mine Equipment Election, International Journal of Mining, Reclamation and Environment 1, 159-165 (1987).

24. N. Seyedaghaee, S. Rahati, H. Alinejad-Rokny, F. Rouhi, An Optimized Model for the University Strategic Planning, International Journal of Basic Sciences & Applied Research, 2(5): 500-505 (2013).

25. S. Anon, Belt Conveyors for Bulk Materials. Conveyor Equipment Manufacturers Association, 5the International Conference on Dam Engineering, Lisbon (2007).

26. Y. Chang, Applications of the Extent Analysis Method on Fuzzy AHP, European Journal of Operational Research 95, 649-655 (1996).

27. L. Zare, and M. Ataei, The Application of Fuzzy Analytic Hierarchy Process (FAHP) Approach to Selection of Optimum Underground Mining Method for Jajarm Bauxite Mine, Expert Systems With Applications 36, 8218-8226 (2009).