HIERARCHY OF EMERGENT PROPERTIES OF ALTERNATIVES IN DECISION-MAKING THEORY

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Abstract: *It is shown, that under vector approach a decision-making problem by means of decomposition of alternative's emergent properties can be presented by a hierarchical system of criteria. At the bottom level estimation of the alternative on separate properties is carried out through a vector of criteria, and at the top level by means of mechanism of composition the estimation of the alternative as a whole turns out. The problem is solved by method of nested scalar convolutions of vector-valued criteria. The methodology of the problem solving is based on complementarity principle by N. Bohr and theorem of incompleteness by K. Gödel.*

Key words: *hierarchy, emergence, nested scalar convolutions, multi-criteria problems, decision-making.*

1. Contents of the problem

An important concept of systems analysis is the integrity of the system. Some phenomena can only be understood by studying the whole object of study (the organism, system, etc.) in its entirety. For example, we can not perceive a piece of music by studying only its individual sounds and beats. This phenomenon is defined in science by the term "emergence" (from the English: Emergence – nascence, appearance of new): the existence in a system the integral properties not possessed by its individual elements. The emergence is a criterial feature of the system.

The feature of emergence is non-additivity properties of the system, the non-applicability of the principle of superposition, the nonlinear coupling between the properties of the system and its individual elements. The appearance of emergence is the result of elements synergy (enhancing of properties). In fact, it is nothing else as a well-known dialectical law of the transformation of quantity into quality. The emergence is due to the interaction of elements (subsystems) within the operating system, which as a whole reveals inherent only to it new quality and regularity.

Thus, the system "vision" has such a property as three-dimensional perception, which is not present at one of its elements (left or right eye). This is the basis of technical systems using stereoscopic effect. There are objects which property and coherent properties of the individual elements are diametrically opposed. In Victor Hugo's novel "Les Miserables" is a description of the bridge: every brick in the bridge hanging over the abyss, tends to fall down, but because they want to fall at the same time and all together, you have a strong holistic arched bridge design which holds.

There are three causes of emergence:

1. Systems nonlinearity. Examples: threshold-like exceeding of critical mass of the compound nucleus, leading to chain reaction (atomic bomb); crystallization of the

supersaturated solution; use of catalysts to change the rates of chemical reactions; the occurrence of water from hydrogen and oxygen.

2. Unpredictable bifurcation in the evolution of a subsystem (the appearance of a new branch of a growing tree).

3. Recombination of links between elements. For example, we all know that water vapor is transferred to a liquid at low temperatures, but not everyone knows that in this case the number of intermolecular links increases. Number of molecules in an isolated system remains the same, but the quality (properties) changes dramatically.

Complex systems are characterized by hierarchy of emergent properties. Let us consider some examples from the different subject areas. In physics, the system of a single atom has its own emergent properties, being combined into a new system – a molecule with its properties, then – in substance, etc. At each new level, there is a new system with new emergent properties that did not exist at the lower levels of the system. In the container with a gas at a microscopic level, we have a set of molecules, the position and speed of each of which are constantly changing. At the holistic level, we consider this environment as a whole, ignoring its components. Here we introduce new emergent characteristics of the state of the gas: pressure, temperature, volume. These three “holistic” values are linked by precise laws. Importantly, these characteristics can not be expressed at a lower level (for example, a molecule has no temperature).

In chemistry, often, based on the properties of the individual components of the object, it is impossible to predict the properties of the object as a whole. For example, hydrogen and oxygen combine to produce water, quite unlike the source gases. Carbon in various allotropic modifications may be graphite and diamond.

The emergence is strong in social systems (ant, bee hive, bird flock, crowd, etc.). A bird in flock partially loses its maneuverability, but only a flock is capable of long-distance flight to warmer climes. A man in the crowd loses some of his freedom, but group action is more effective than individual. It is interesting that for the output from the crowd a person needs to spend some energy to overcome the system force.

In biology, the emergent properties appear when viewed at different levels – from the molecular, ending the biosphere. The cell is not a simple union of chemical molecules. The body is not a simple set of cells; the population is not a mechanical set of organisms. Engels: "We shall certainly “reduce” sometime intellection experimentally to molecular and chemical motions in the brain, but does this exhaust the essence of thought?” Psyche is an emergent property of the holistic structure of a higher order – the nervous system.

Finally, in decision theory, the integral properties of an alternative undergo decomposition, resulting in a hierarchical structure of emergent properties (analysis). Any multiobjective problem can be represented by a hierarchical system, at the lower level of which the individual properties of the object are assessed using the vector criteria, and the evaluation of the whole object is obtained at the top level by mechanism of the composition (synthesis). Central here is the problem of the composition of criteria for levels of the hierarchy.

2. Multi-criteria decision-making problem

The problem of decision-making in general view [Gubanov at al, 1988] can be represented by the scheme

$$\{\{x\},Y\} \to x^* \ .$$

where $\{x\}$ – a set of objects (alternatives); Y – function of choice (rule establishing a preference on the set of alternatives); x^* – selected alternatives (one or more).

The set $\{x\}$ can be discrete (example: several projects of the aircraft from which you must choose the best) or continuous (range of radio adjuster, from which the tuning is selected to the correct channel).

Function Y is used to solve the problem of analysis and evaluation of alternatives. The result of evaluation should be the choice of one or more of the best alternatives in a given set. In decision theory, there are two approaches to the assessment of objects (alternatives) to be selected.

One of them – estimation of the object as a whole and the choice of an alternative by means of the direct comparison of objects as gestalts (holistic images of objects without detailing their properties). A notorious example is estimation of an actor's performance by K. Stanislavsky: "I believe!". It is clear that a holistic approach is openly subjective, based on the individual preferences of the decision maker (DM) and absolutely can not be formalized. Takes place a dichotomy in the selection of alternatives: "like" - "do not like". If there arises a question – *why* do you like (or dislike), then you should use the second approach to the analysis and evaluation of alternatives.

The second approach – detailing and evaluation of various vectors of emergent properties of objects and making decisions based on the comparison of these properties. If a holistic approach provides choice $x*$ directly on the selection function Y, the vector approach requires a mechanism to carry out decomposition of Y on aggregate (vector) of the functions y. Under *decomposition* of choice function Y is understood its equivalent representation using a certain set of other choice functions y, *composition* of which is the original choice function Y.

The modern line in the theory of decision-making consists in use of the vector approach. It is explained by its objectivity and universality, and also basic opportunity of application of the formalized methods. It is taken into account also concreteness and clearness of the approach, as on a narrow question there are less divergences in opinions, it is easier to collect the indisputable facts.

It is assumed that in respect of an individual property is much easier to tell which of the alternatives is preferable for the decision maker. For example, in the task of selecting the best aircraft project we can say much more confidently that the project A is better than the project B *by the property* of comfort, or reliability, or capacity, rather than the fact that the project A is better than the project B *as a whole*. Separation of properties of alternatives on the basis of the analysis is the decomposition leading to the hierarchical structure of properties. Properties of the first hierarchical level can be subdivided into the following sets of properties, etc. The dividing depth is determined by the desire to reach those properties, which are convenient for comparing with each other.

Indeed, in the example of the airplane to judge on comfort, of course, is easier than on the airplane as a whole, but such a qualitative property is not very convenient for comparison and requires further decomposition for convenience and objectivity of properties comparison. Therefore the comfort property, in turn, undergoes decomposition to: a) the noise level in the cabin, b) the level of floor vibration, and c) the distance between the seats, etc. These characteristics are expressed in numbers and are objective.

Properties, for which there exist objective numerical characteristics, are called *criteria*. More rigorously: the criteria are called quantitative properties of the object, the numerical values of which are a measure of the quality assessment of the object in relation to this property. Getting a set of criteria – this is the final result of the hierarchical decomposition. The amount of levels depends on the desired depth of decomposition. The difficulty lies in the fact that for each of the initial properties the depth of the decomposition can be various, and at each hierarchical level is necessary

to normalize the sets of heterogeneous criteria.

The approach of comparison on separate properties, at all its attraction, derivates a serious problem of return transition to required comparison of alternatives as a whole. This problem involves the solution of the problem of criteria *composition* for levels of the hierarchy, which is quite difficult, especially at a considerable depth of properties decomposition. In the simplest and most widespread case (two-level hierarchy), the problem is solved in traditional form as a single scalar convolution of criteria, the numerical value of which appears as an estimate of the quality of the object (alternative) as a whole. But then in the presence of a three-level hierarchy other approaches are required.

The foregoing gives reason to believe that any multicriteria problem can be represented by a hierarchical system, on the lower level of which the evaluation of individual properties of the object using a vector of criteria takes place, and on the upper level, through the mechanism of the composition an estimate of the object as a whole is obtained. Central here is the problem of the composition of criteria for levels of the hierarchy.

3. Statement of the problem

Quality of an alternative is determined by hierarchical system of vectors

$$y^{(j-1)} = \{y_i^{(j-1)}\}_{i=1}^{n^{(j-1)}}, \ j \in [2, m]$$

where $y^{(j-1)}$ is the vector of criteria on the (*j*-1)-th level of the hierarchy, by the components of which the quality of properties of alternatives for the *j*-th level is assessed; *m* is the number of levels of the hierarchy; $n^{(j-1)}$ is the number of estimated properties on (*j*-1)-th level of the hierarchy. The numerical values of *n* criteria $y^{(1)} = y$ of the first level of the hierarchy for the alternative are given.

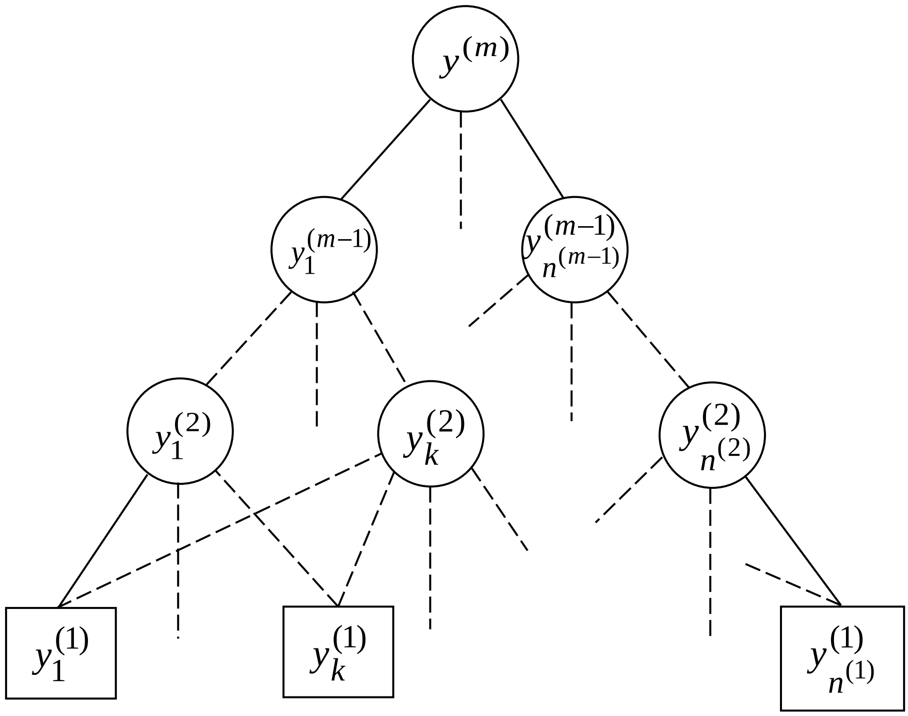

Fig.1

The same criterion on (j-1)-th level can participate in the evaluation of several properties of the j-th level, i.e. in the hierarchy are possible cross-links. The block diagram of an alternative system of quality criteria is shown in Figure 1. It is clear that $n^{(1)} = n$ and $n^{(m)} = 1$.

Importance (significance) of each of the components of the criterion of (j-1)-th level in the evaluation of properties of k-th level is characterized by a coefficient of the priority, their set forming the priority vectors system

$$p_{ik}^{(j-1)} = \{p_{ik}^{(j-1)}\}_{k=1}^{n^{(j)}},\ j \in [2, m].$$

It is required to find an analytical evaluation y^* and a qualitative evaluation of the effectiveness of the alternative, and from available alternatives to choose the best.

4. The method of solution

To solve this problem a systemic approach is used in which each of the alternatives (objects) is regarded as a set of elements with different emergent (including contradictory) properties different from those of the whole system.

Compare the said with the principle of complementarity, introduced into science by Niles Bohr: "... To reproduce the integrity of the phenomenon should be used mutually exclusive "complementary" classes of concepts, each of which can be used in its own, special conditions, but only when taken together, exhaust the definable information." For a complete description of the object they are equally necessary and therefore do not contradict, but complement each other.

Multiple properties of a complex system in a given situation of its functioning are evaluated quantitatively by relevant partial criteria. In different situations the rank «most important» acquire different properties and, consequently, different partial criteria. Thus, mutually exclusive "complementary" classes of concepts which appear as individual theoretical models are characterized by partial contradictory criteria, each of which is most useful in its own, special conditions. It is the principle of complementarity that allows for separating and then linking these criteria in multicriteria evaluation. Only a full set of individual criteria (vector criterion) enables an adequate assessment of the functioning of a complex system as manifestation of the contradictory unity of all its properties.

However, this possibility represents only a necessary but not a sufficient condition for a vector evaluation of the entire alternative as a whole. Indeed, let it be that at the lower level of the hierarchy of criteria the numerical values of partial criteria of comfort properties of aircraft are known, such, as the distance between the seats, the noise level in the cabin, the amplitude of the vibration of the floor, etc. Does it mean that we, knowing these values, can estimate the property of comfort as a whole? No, we can not.

It is appropriate to recall the old Indian parable about the blind men who wished to become familiar with an elephant. One touched the trunk and decided that the elephant is similar to a snake. The second picked up the ear and told that the elephant reminds to him a bed-sheet. The third felt the leg and declared that the elephant is a pole. These individual elephant "models" reflect the various properties of the object, but do not give the whole picture.

For a complete evaluation it is necessary to go out from the lower level of the hierarchy and to rise on the following tier, i.e. to carry out an act of criteria composition. Let's compare this with the incompleteness theorem of Kurt Gödel "... In every complex enough not contradictory theory of the first order there is a statement, which by means of the theory is impossible neither to prove, nor to deny. But the self-consistency of a particular theory can be established by means of another, more powerful formal theory of the second order. But then the question of the self-consistency of this second theory arises, and so forth.” We can say that Gödel’s theorem is a methodological basis for the study of hierarchical structures.

With reference to our problem it means that for an adequate estimation of an alternative as a whole we should solve a task of the criteria composition on levels of hierarchy, consecutively passing from the bottom level up to top.

A scalar convolution of criteria can serve as a tool for the act of composition. The scalar convolution – it is a mathematical technique for data compressing and quantifying its integral properties by a single number. In [Voronin, 2014], a scalar convolution on **nonlinear compromise scheme** (NCS) for the criteria subject to be minimized is proposed

$$Y[y(x)] = \sum_{k=1}^{s} p_k A_k \left[A_k - y_k(x)\right]^{-1}$$

applied in cases where the decision-maker considers as preferred those solutions in which the values of individual criteria $y_{\kappa}(x)$ are furthest from their limit values, A_{κ}. This convolution has a number of essential advantages, which include flexibility, universality and analyticity.

The choice of a compromises scheme is made by the DM and appears as explicitly conceptual [Fishburne, 1978].

In the problem of making the choice the amount of variants (alternatives) is $n_a \geq 1$. Each variant is characterized by its own hierarchical structure. If $n_a = 1$, the problem is transformed to the task of evaluation of this given hierarchical structure. If $n_a > 1$, each structure is estimated as a given one and that option is chosen, the hierarchical structure of which gets the best estimate. Therefore, when a discrete multiobjective optimization takes place, as a basic here the problem of estimating a given hierarchical structure is considered. However, we do so only in the case of a relatively small amount of alternatives n_a , when the method of simple enumeration does not cause significant computational difficulties. When large volumes of sets of alternatives take place, we should employ other methods of optimization, such as described in [Voronin, 2014].

Evaluation of this given alternative is nothing else but a solution of the problem of analysis of the alternative quality under a given argument $x^{(o)}$ from the set $\{x\}$. This enables henceforth in the expressions of criteria to not include the values of the argument x.

Nested scalar convolutions. It is proposed for analytical evaluation of hierarchical structures to apply a method of nested scalar convolutions. The composition is performed on the “matryoshka principle”: the scalar convolutions of the weighted components of vector criteria of lower level serve as the components of the vectors of higher level criteria. Scalar convolution of criteria obtained at the uppermost level is automatically considered as the expression for the analytical evaluation of effectiveness of the entire hierarchical system.

The algorithm for nested scalar convolutions is represented by an iterative sequence of operations of the weighed scalar convolutions of criteria for each level of the hierarchy from the bottom up, taking into account the priority vectors, based on the selected compromise scheme

$$\{(y^{(j-1)}, p^{(j-1)}) \to y^{(j)}\}_{j\in[2,m]} \quad (1)$$

and the searching and evaluating of effectiveness of the entire hierarchical system (alternative) as a whole is expressed by the problem of determining the scalar convolution of criteria on the top level of the hierarchy:

$$y^* = y^{(m)}.$$

When using the recurrent formula (1) important is the rational choice of the compromise scheme. For the method of nested scalar convolutions the adequate is the NCS. It is established that, without loss of generality, a premise for its use is that all the partial criteria were non-negative, were subject to minimization and were limited:

$$0 \le y_i \le A_i, A = \{A_i\}_{i=1}^{n},$$

where A is the vector of restrictions on the criteria of the current level of the hierarchy; n is the amount of them.

Proceeding from (1) the expression to evaluate k-th property of an alternative for the j-th level of the hierarchy by using the nonlinear compromise scheme looks like

$$y_k^{(j)} = \sum_{i=1}^{n_k^{(j-1)}} p_{ik}^{(j-1)}[1 - y_{0ik}^{(j-1)}]^{-1}, k \in [1, n^{(j)}], \quad (2)$$

where criteria of the (j-1)-th level are normalized (reduced to unity). Thus, $y_{0ik}^{(j-1)}$ are the normalized vector's $y_0^{(j-1)}$ components involved in the evaluation of properties of the k-th alternative on the *j*-th level of the hierarchy; $n_k^{(j-1)}$ is their amount; $n^{(j)}$ is the amount of evaluated properties of the *j*-th level.

Coefficients of priority *p* are the formal parameters with dual physical meaning. On the one hand, they are priority coefficients expressing the preferences of individual decision-makers concerning certain criteria. On the other – they are the regression coefficients of the model constructed on the basis of the concept of nonlinear compromise scheme. Determination of the coefficients *p* at each level of the hierarchy can be done by optimizing on the simplex using a dual approach described in [Voronin, 2014], or by expert analysis using an ordinal (serial) or a cardinal (interval) scale.

In the most simple and rather common case the multicriteria problem is formulated and solved without priorities, when decision-makers believe that all the importance parameters for all properties of alternatives are the same. In this case, a simple scalar convolution with the nonlinear trade-offs scheme in a unified form is used [Voronin, 2014].

The recurrent formula for calculating the criteria. In order to formula (2) reflected the idea of the nested scalar convolutions method in accordance with the recurrent relation (1), this expression should be normalized, i.e., must be obtained a relative measure such that it were subject to be minimized, and it were the unit for it as the limit value.

The structure of the nonlinear compromise scheme enables normalizing the convolution (2) not to the maximum (which in this case is difficult), but to the minimum value of criteria convolution. Indeed, the ideal values for the criteria that are subject to be minimized are their zero points. Putting in (2)

$$y_{0ik}^{(j-1)} = 0, \forall i \in [1, n_k^{(j-1)}]$$

and taking into account the normalization $\sum_{i=1}^{n} p_i = 1$, we obtain $y_{k\min}^{(j)} = 1$. After calculations [Voronin, 2014], the final expression for the recurrent formula for calculating analytical assessments of the alternatives properties at all levels of the hierarchy becomes

$$y_{0k}^{(j)} = 1 - \{ \sum_{i=1}^{n_k^{(j-1)}} p_{ik}^{(j-1)} [1 - y_{0ik}^{(j-1)}]^{-1} \}^{-1}, k \in [1, n^{(j)}], j \in [2, m] \qquad (3)$$

5. Qualitative evaluation of alternatives

A qualitative (linguistic) estimate is obtained by comparing the analytic evaluation of an alternative with the inverse normalized fundamental scale. The general concept of an ordinal fundamental scale is described in [Saaty, 1990]. An interval normalized inverted scale is presented by the Table 1.

Table 1. Fundamental scale

Quality category	**Ranges of normalized inverse fundamental scale for**
Unacceptable	1,0 – 0,7
Low	0,7 – 0,5
Satisfactory	0,5 – 0,4
Good	0,4 – 0,2
High	0,2 – 0,0

It shows the relationship between qualitative gradations of properties of objects and the corresponding normalized quantitative estimates y_0. We can say that in terms of the theory of fuzzy sets [Saaty, 1990], the fundamental scale acts as a universal membership function for the transition from the digital quantity to the appropriate quality grading and back. The transition from the linguistic variable estimate (satisfactory quality, high quality, etc.) to the appropriate quantitative estimates on the rating scale is carried out, i.e. a transition from fuzzy quality grades to numbers and vice versa takes place.

Evaluating variants using a unified normalized fundamental scale makes it possible to solve multicriteria problems both in traditional formulations and in the case where an alternative should be selected from a set of inhomogeneous alternatives, for which a unified set of quantitative assessment criteria cannot be formulated, and to estimate the unique alternative.

An illustrative example of calculation of emergent properties of a hierarchical system of specific alternative (draft of a plane) is given.

6. Illustrative example

It is required to find a quantitative $y_0^* = y_0^{(3)}$ and qualitative evaluation of aircraft project for two main characteristics: comfort, characterized by evaluation criterion $y_{01}^{(2)}$ and reliability, which is mapped as evaluation criterion $y_{02}^{(2)}$. Block diagram of a three-level hierarchy of criteria to evaluate the project is presented in Figure 2.

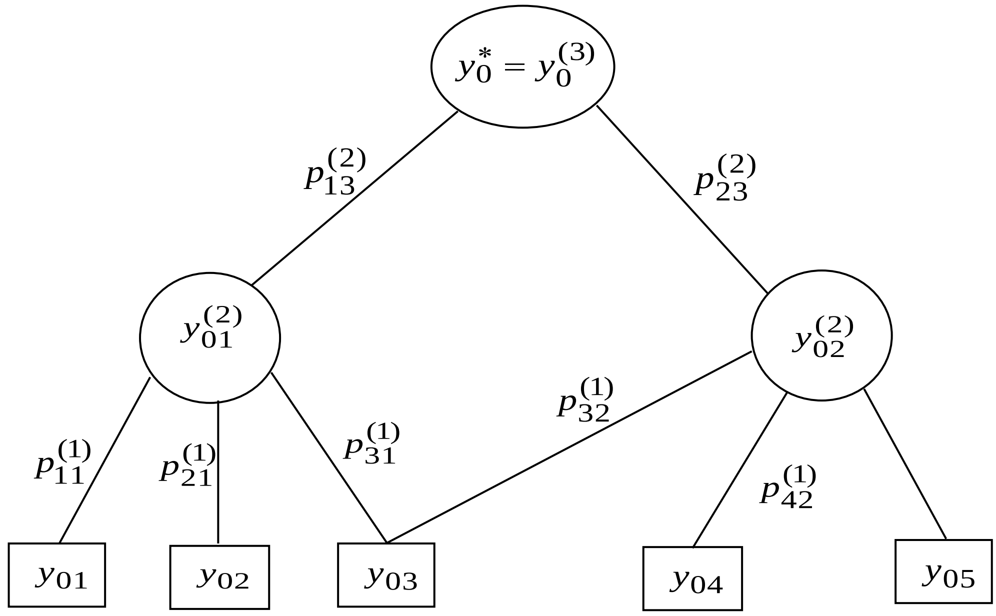

Fig. 2

The following numeric values are given. Criteria of lower (first) hierarchy level: the distance between the seats in the passenger cabin y_{01} = 0.3, noise level in the cabin y_{02} = 0.5, vibration level in the cabin floor y_{03} = 07, probability of equipment failure y_{04} = 0.2, structural strength y_{05} = 0.1. Priority coefficients: $p_{11}^{(1)}$ = 0.7, $p_{21}^{(1)}$ = 0.2, $p_{31}^{(1)}$ = 0.1, $p_{32}^{(1)}$ = 0.1, $p_{42}^{(1)}$ = 0.45, $p_{52}^{(1)}$ = 0.45, $p_{13}^{(2)}$ = 0.5, $p_{23}^{(2)}$ = 0.5.

Using the recurrent formula (3), we obtain the analytic evaluation of comfort property

$$y_{01}^{(2)} = 1 - \frac{1}{0{,}7\frac{1}{1-0{,}3} + 0{,}2\frac{1}{1-0{,}5} + 0{,}1\frac{1}{1-0{,}7}} = 0{,}42\,.$$

Comparing this with the analytic grading of the Table 1, we find that the comfort property of the aircraft project quality is *satisfactory*.

An analytic expression for the evaluation the reliability property is

$$y_{02}^{(2)} = 1 - \frac{1}{0{,}1\frac{1}{1-0{,}7} + 0{,}45\frac{1}{1-0{,}2} + 0{,}45\frac{1}{1-0{,}1}} = 0{,}28.$$

According to Table 1, the quality of the reliability property of this project is estimated as *high*.

At the final (second) stage of criteria the composition formula (3) gives the result

$$y_0^{*} = 1 - \frac{1}{0{,}5\frac{1}{1-0{,}42} + 0{,}5\frac{1}{1-0{,}28}} = 0{,}36.$$

The Table 1 shows that with this analytic assessment of the aircraft project its quality as a whole is evaluated as *good.*

7. Conclusion

The foregoing leads to the conclusion that any problem of vector evaluation alternatives can be represented by a hierarchical system of criteria resulting from the decomposition of emergent properties of the alternatives. At the lower level of the hierarchy the object (alternative) is being evaluated by the individual properties using the initial criteria vector, and at the top level evaluation of the whole object is obtained by the mechanism of composition. Central here is the problem of the composition of criteria for levels of hierarchy to be solved by nested scalar convolutions method.

The methodological basis of the *decomposition* of properties of an alternative to obtain the initial vector of criteria (multicriteriality) is the principle of complementarity of N. Bohr. This is *necessary* condition for the vector evaluation of alternatives.

Methodology of criteria *composition* for levels of the hierarchy is based on the Gödel's theorem of incompleteness. This is *sufficient* condition for the vector evaluation of alternatives.

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